Advanced Computer Graphics
Materials 1

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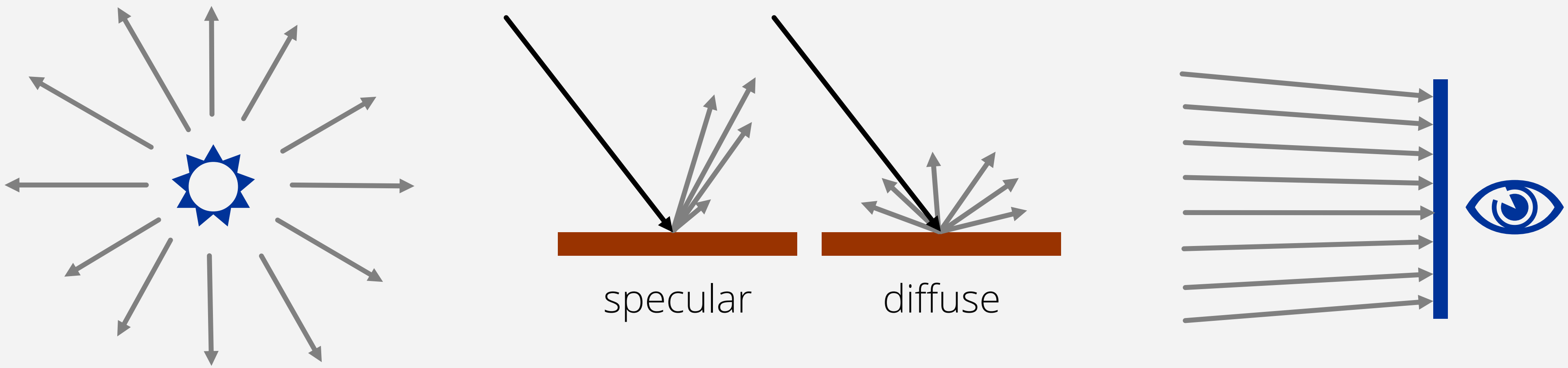
UNI FREIBURG

Outline

- Context
- Bidirectional Reflectance Distribution Function BRDF
  - Definition
  - Application
  - Exemplary materials
  - Properties
- Reflectance
- Material modeling
- Transparent materials

Context



Light is emitted at light sources

Light is absorbed and scattered at surfaces

Cameras capture light

- Light absorption and scattering at surfaces is governed by material properties
- How to represent material properties?

Surface Reflection Properties

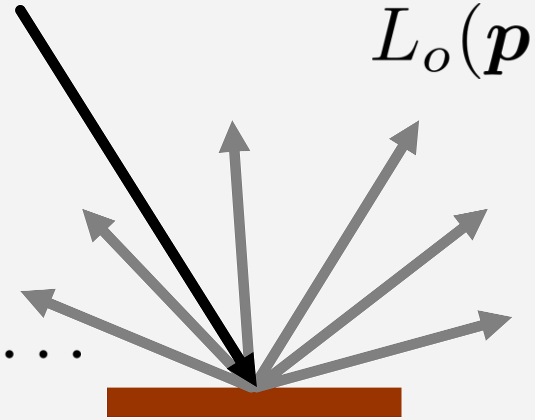
– How much incident light from a particular direction is reflected into a particular direction?
⇨ Bidirectional Reflectance Distribution Function BRDF f_r

$L_i(\boldsymbol{p}, \boldsymbol{\omega}_i)$

$$L_o(\boldsymbol{p}, \boldsymbol{\omega}_o) \sim f_r(\boldsymbol{p}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o) L_i(\boldsymbol{p}, \boldsymbol{\omega}_i)$$

Incoming light
from direction $\boldsymbol{\omega}_i$

Outgoing light into direction $\boldsymbol{\omega}_o$



$L_o(\boldsymbol{p}, \boldsymbol{\omega}_o) = \dots$

$L_o(\boldsymbol{p}, \boldsymbol{\omega}_o) = \dots$

Position $\boldsymbol{p}$

Surface Reflection Models

- Surfaces have reflection properties
  - How much flux is absorbed?
  - Which part of the flux is reflected?
  - How is it reflected?
- E.g., empirical Phong model
  - Efficient to compute
  - Physically motivated
  - Does not capture all aspects of real materials (too few degrees of freedom)

Surface Reflection Models

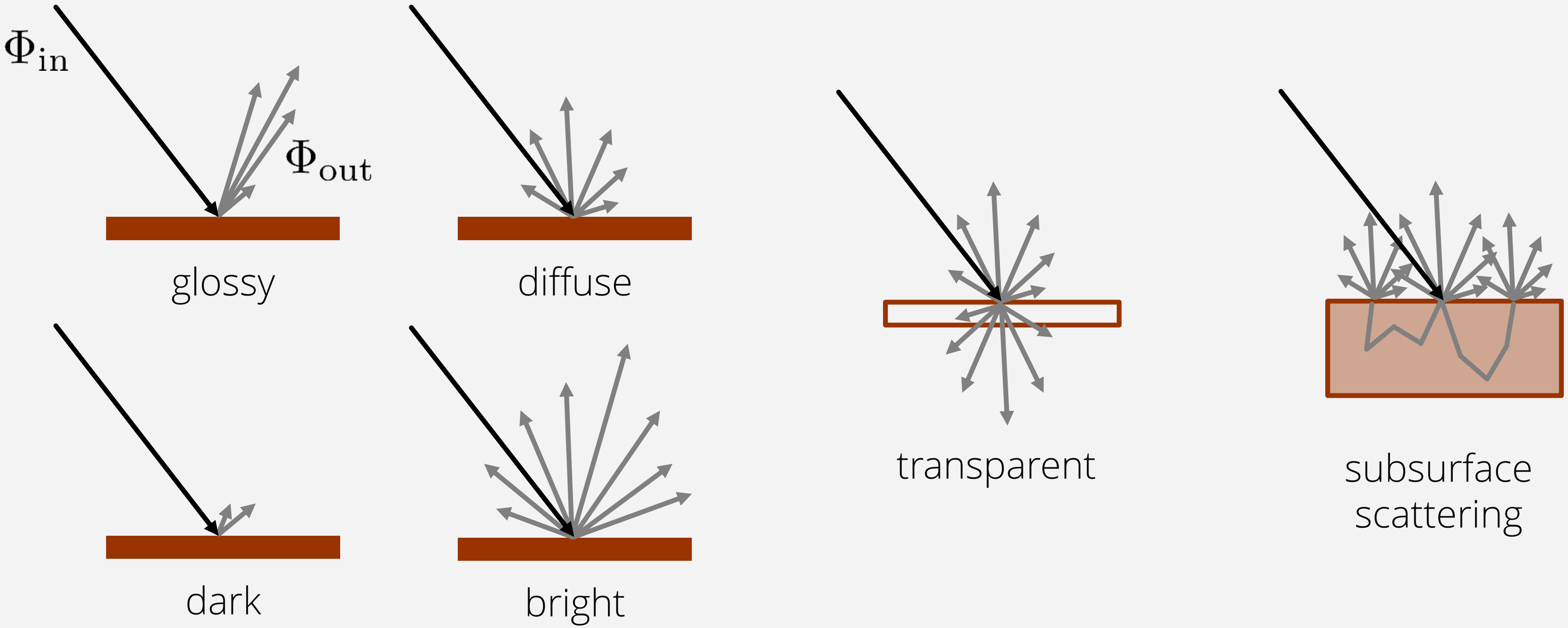
- Theoretical reflectance models
  - Have more degrees of freedom
- Theoretical models can consider
  - Incoming and outgoing angle of flux
  - Incoming and outgoing wavelength of flux (fluorescence)
  - Incoming and outgoing polarization (linear and circular)
  - Incoming and outgoing position (subsurface scattering)
  - Time delay between incoming and outgoing flux (phosphorescence)

Surface Reflection Models

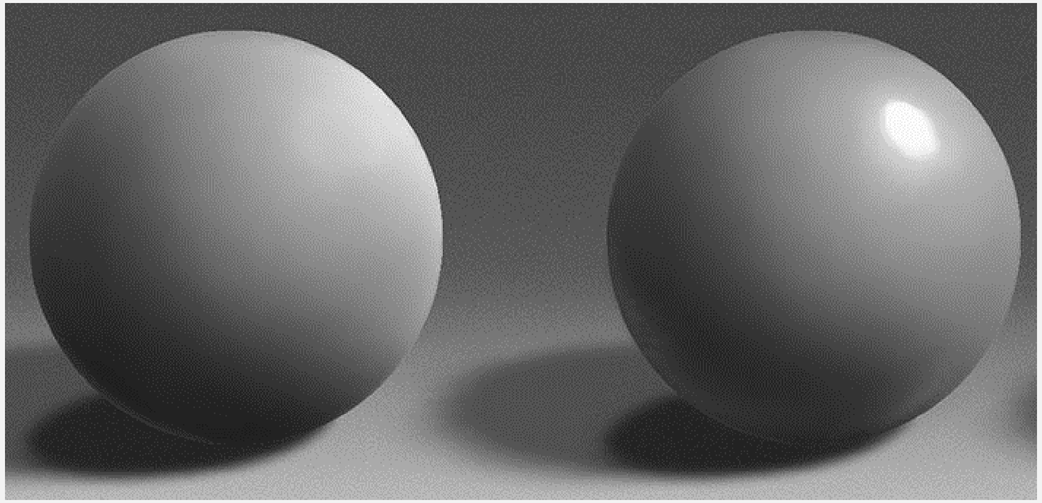
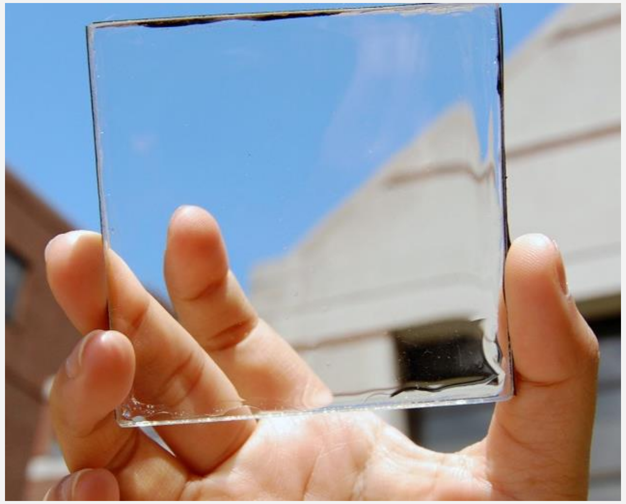
- Incident light is partially reflected or absorbed depending on the absorbance spectrum of the surface (surface color)
- Reflected light is scattered by the surface depending on the surface roughness
  - Smooth surface (small patches with uniform orientation)
    - Specular or glossy reflection
    - Light is reflected into dominant reflection directions
  - Rough surface (small patches with varying orientation)
    - Diffuse (Lambertian) reflection
    - Light is reflected into many directions

Materials

– Can be described by relating incident and exitant flux



Materials


diffuse glossy transparent subsurface scattering

[Oliver Wetter]

[David Turesson]

[https://cgiknowledge.wordpress.com/]

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Introduction

– Incident radiance $L_i(\boldsymbol{p}, \boldsymbol{\omega}_i)$
at position $\boldsymbol{p}$ from direction $-\boldsymbol{\omega}_i$
induces irradiance at $\boldsymbol{p}$:

$$\mathrm{d}E_i(\boldsymbol{p}, \boldsymbol{\omega}_i) = L_i(\boldsymbol{p}, \boldsymbol{\omega}_i) \cos\theta_i \mathrm{d}\omega_i$$

– Flux is partially absorbed:

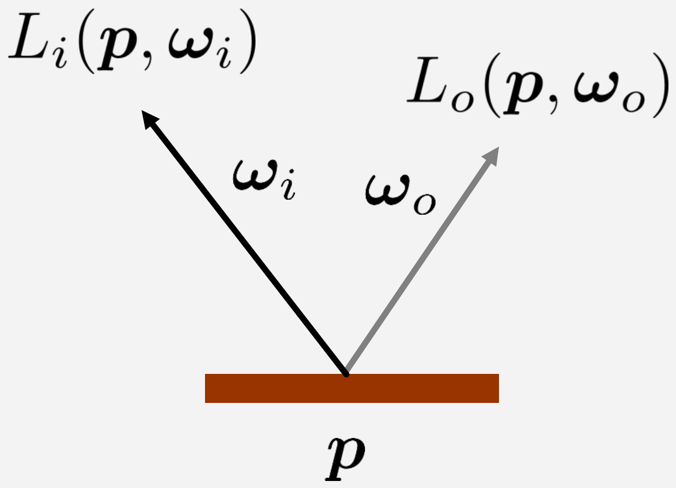
$$\mathrm{d}B_i(\boldsymbol{p}, \boldsymbol{\omega}_i) = \rho(\boldsymbol{p}) \mathrm{d}E_i(\boldsymbol{p}, \boldsymbol{\omega}_i)$$

$0 \leq \rho \leq 1$ is a reflectance coefficient.

– Reflected flux into direction $\boldsymbol{\omega}_o$

$$\mathrm{d}L_o(\boldsymbol{p}, \boldsymbol{\omega}_o) \sim \mathrm{d}B_i(\boldsymbol{p}, \boldsymbol{\omega}_i) \sim \mathrm{d}E_i(\boldsymbol{p}, \boldsymbol{\omega}_i)$$

Reflection pattern   Reflectance



ω_i represents the direction of the incident radiance. Per definition, all directions point away from the surface. I.e., incident radiance travels along $-\omega_i$.

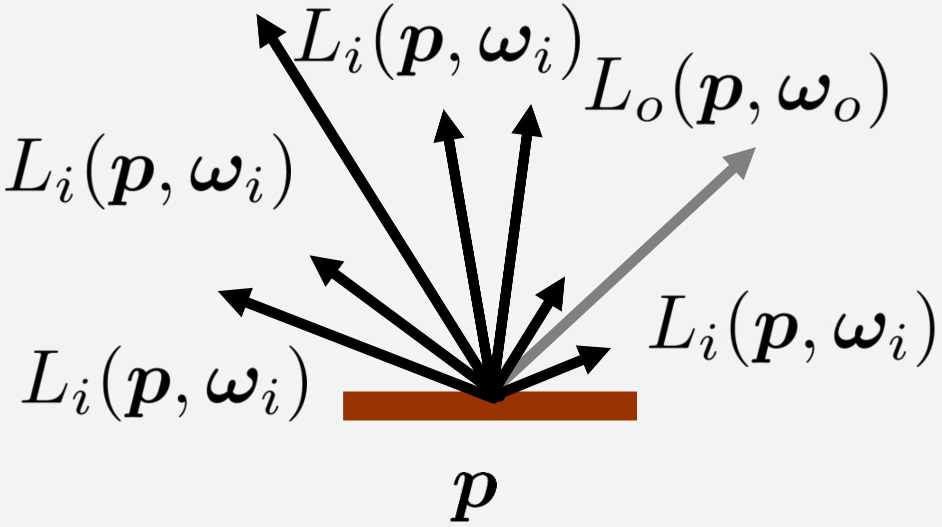
Definition

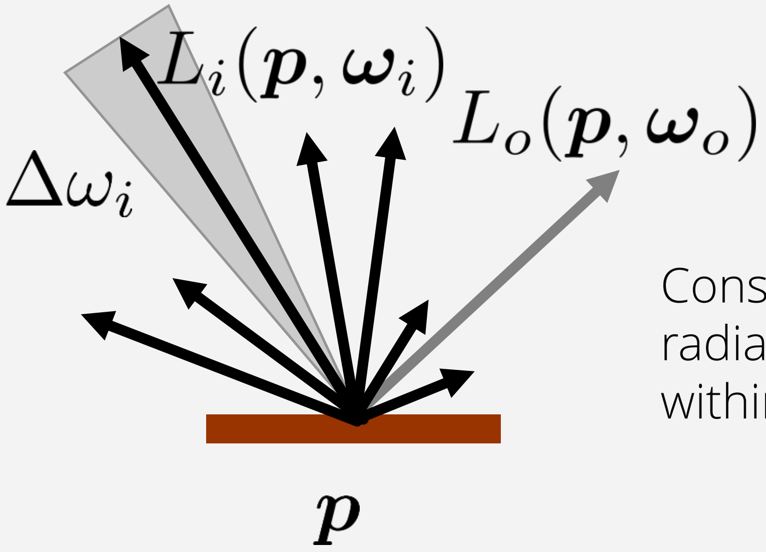
- For all pairs of directions $\boldsymbol{\omega}_i$ and $\boldsymbol{\omega}_o$, the ratio of outgoing radiance towards $\boldsymbol{\omega}_o$ and irradiance due to incoming radiance from $-\boldsymbol{\omega}_i$ is referred to as BRDF: $f_r(\boldsymbol{p}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \frac{\mathrm{d}L_o(\boldsymbol{p},\boldsymbol{\omega}_o)}{\mathrm{d}E_i(\boldsymbol{p},\boldsymbol{\omega}_i)}$
- BRDF typically depends on a position and two directions
  - Directions form a solid angle of 2π for opaque surfaces and 4π for transparent surfaces
  - Various variants. E.g., BRDF can depend on two positions for subsurface scattering $f_r(\boldsymbol{p}_i, \boldsymbol{p}_o, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \frac{\mathrm{d}L_o(\boldsymbol{p}_o,\boldsymbol{\omega}_o)}{\mathrm{d}E_i(\boldsymbol{p}_i,\boldsymbol{\omega}_i)}$

Application

- Relation between irradiance and exitant radiance
  $$\mathrm{d}L_o(\boldsymbol{p}, \boldsymbol{\omega}_o) = f_r(\boldsymbol{p}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o)\mathrm{d}E_i(\boldsymbol{p}, \boldsymbol{\omega}_i)$$
- Irradiance is induced by radiance
  $$\mathrm{d}L_o(\boldsymbol{p}, \boldsymbol{\omega}_o) = f_r(\boldsymbol{p}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o)L_i(\boldsymbol{p}, \boldsymbol{\omega}_i)\cos\theta_i \mathrm{d}\omega_i$$
- Integration over the hemisphere ⇨ reflectance equation
  $$L_o(\boldsymbol{p}, \boldsymbol{\omega}_o) = \int_{2\pi} f_r(\boldsymbol{p}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o)L_i(\boldsymbol{p}, \boldsymbol{\omega}_i)\cos\theta_i \mathrm{d}\omega_i$$
- Reflectance equation establishes a relation between incident and exitant radiance

Reflectance Equation





$$L_o(\boldsymbol{p}, \boldsymbol{\omega}_o) = \int_{2\pi} f_r(\boldsymbol{p}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o) L_i(\boldsymbol{p}, \boldsymbol{\omega}_i) \cos\theta_i \mathrm{d}\omega_i$$

$$L_o(\boldsymbol{p}, \boldsymbol{\omega}_o) \approx \sum_i f_r(\boldsymbol{p}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o) L_i(\boldsymbol{p}, \boldsymbol{\omega}_i) \cos\theta_i \Delta\omega_i$$

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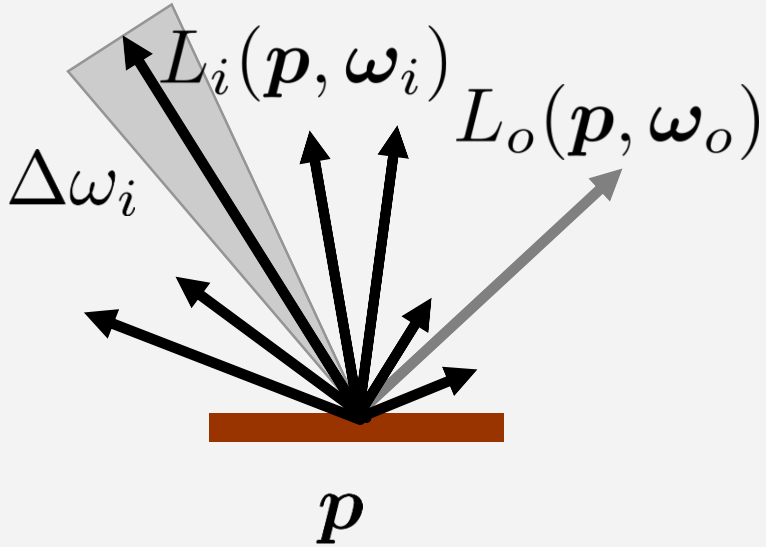
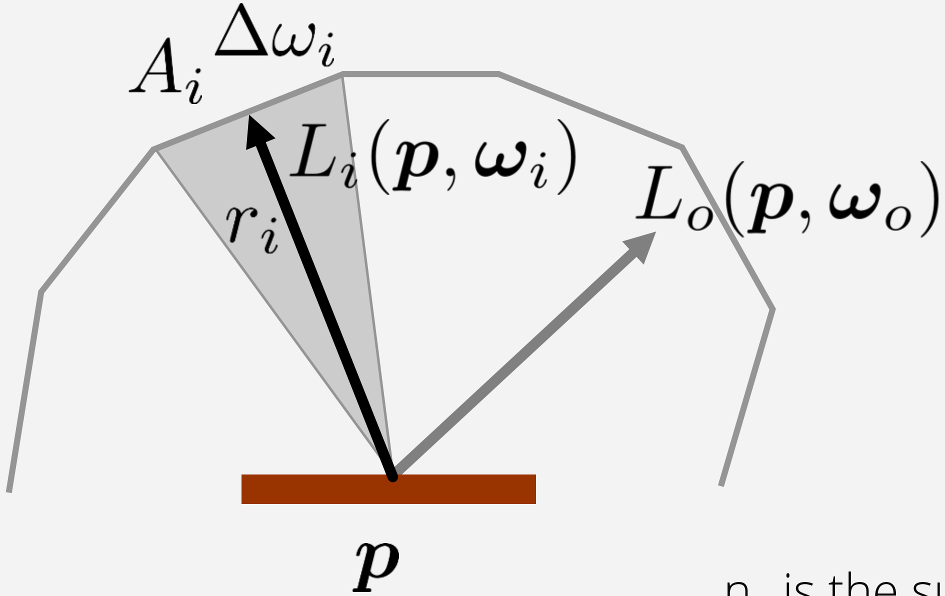
Reflectance Equation



$$L_o(\boldsymbol{p}, \boldsymbol{\omega}_o) \approx \sum_i f_r(\boldsymbol{p}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o) L_i(\boldsymbol{p}, \boldsymbol{\omega}_i) \cos\theta_i \Delta\omega_i$$



$$\Delta\omega_i \approx \frac{\cos(-\boldsymbol{\omega}_i, \boldsymbol{n}_{A_i})}{r_i^2} A_i$$

$$L_o(\boldsymbol{p}, \boldsymbol{\omega}_o) \approx \sum_i f_r(\boldsymbol{p}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o) L_i(\boldsymbol{p}, \boldsymbol{\omega}_i) \frac{\cos(\boldsymbol{\omega}_i, \boldsymbol{n}_p)\cos(-\boldsymbol{\omega}_i, \boldsymbol{n}_{A_i})}{r_i^2} A_i$$

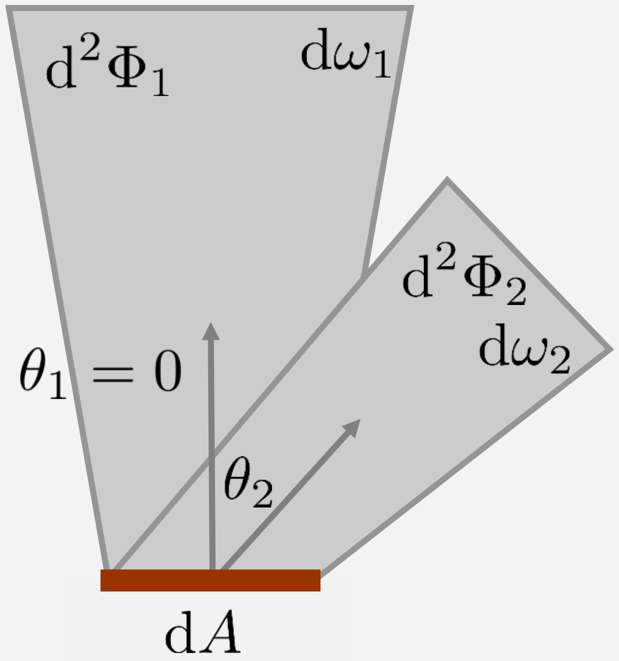
n_P is the surface normal at p.
n_{Ai} is the surface normal at A_i.

Example - Diffuse Reflection

- Lambertian model
  - Def: Reflected flux is proportional to the cosine of the angle between flux direction and surface normal
  - $\Phi \sim \cos\theta$
  - $\Phi^{\perp} = \frac{\Phi_1}{\cos\theta_1} = \frac{\Phi_2}{\cos\theta_2}$
  - Outgoing radiance is equal in all directions

$$L_1 = \frac{\mathrm{d}^2\Phi_1}{\mathrm{d}\omega_1 \cdot \cos\theta_1 \cdot \mathrm{d}A} = \frac{\mathrm{d}^2\Phi_2}{\mathrm{d}\omega_2 \cdot \cos\theta_2 \cdot \mathrm{d}A} = L_2$$

$d\omega_1 = d\omega_2$ denote two solid angles of the same size with different directions.



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Example - Diffuse Reflection

- From a viewer's perspective
  - More flux per surface area is sent towards viewer 1

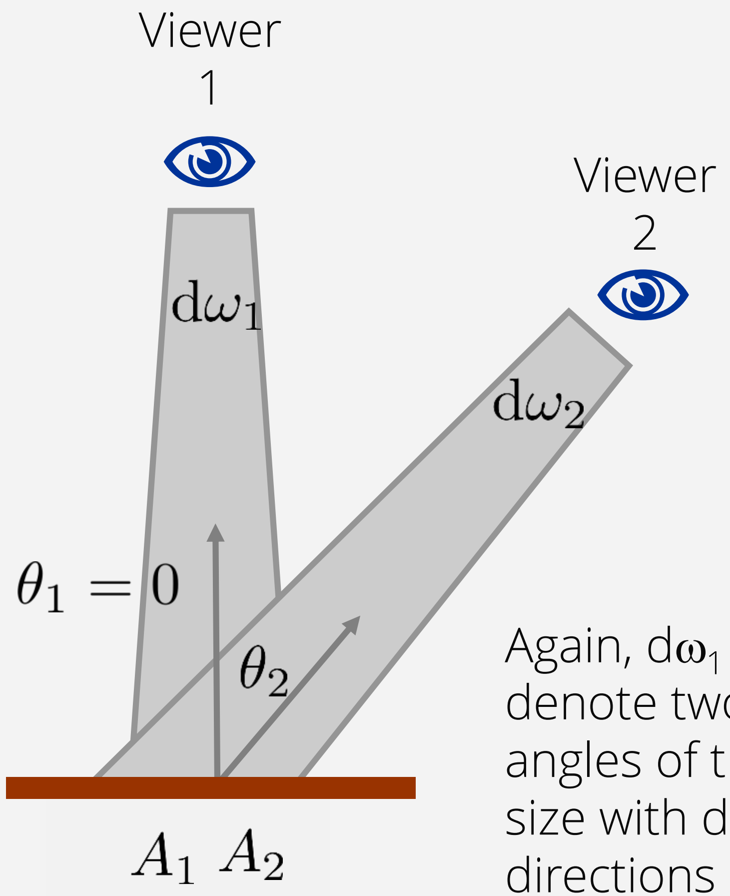
$$\Phi^{\perp} = \frac{\Phi_1}{\cos\theta_1} = \frac{\Phi_2}{\cos\theta_2} = \frac{\Phi_i}{\cos\theta_i}$$

  - Viewer 2 receives flux from (sees) a larger surface area

i denotes an arbitrary direction

$$A^{\perp} = A_1 \cdot \cos\theta_1 = A_2 \cdot \cos\theta_2 = A_i \cdot \cos\theta_i$$

  - Both effects cancel which results in the same flux and radiance towards both viewers



Again, $d\omega_1 = d\omega_2$ denote two solid angles of the same size with different directions

BRDF for Diffuse Reflecting Material

- Illumination $L_i(\boldsymbol{\omega}_i)$
- Induced surface irradiance $\mathrm{d}E_i(\boldsymbol{\omega}_i) = L_i(\boldsymbol{\omega}_i) \cdot \cos\theta_i \cdot \mathrm{d}\omega_i$
- Overall irradiance $E = \int_{2\pi} L_i(\boldsymbol{\omega}_i) \cdot \cos\theta_i \cdot \mathrm{d}\omega_i$
- Partially absorbed. Resulting radiosity

$B = \rho \cdot E \qquad 0 \leq \rho \leq 1$ ρ - reflectance

$B = \int_{2\pi} L_o(\boldsymbol{\omega}_o) \cdot \cos\theta_o \cdot \mathrm{d}\omega_o = L_o \cdot \int_{2\pi} \cos\theta_o \mathrm{d}\omega_o = L_o \cdot \pi$ see next slide

$\rho \cdot E = \pi \cdot L_o \qquad L_o = \frac{\rho}{\pi} E = \int_{2\pi} \frac{\rho}{\pi} \cdot L_i(\boldsymbol{\omega}_i) \cdot \cos\theta_i \cdot \mathrm{d}\omega_i$

$\Rightarrow f_{r,d}(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \frac{\rho}{\pi}$ BRDF is constant for diffuse reflecting material

Integrating Over Solid Angles

- Directions can be represented with two angles
- Differential area spanning from θ to $\theta+d\theta$ and from ϕ to $\phi+d\phi$

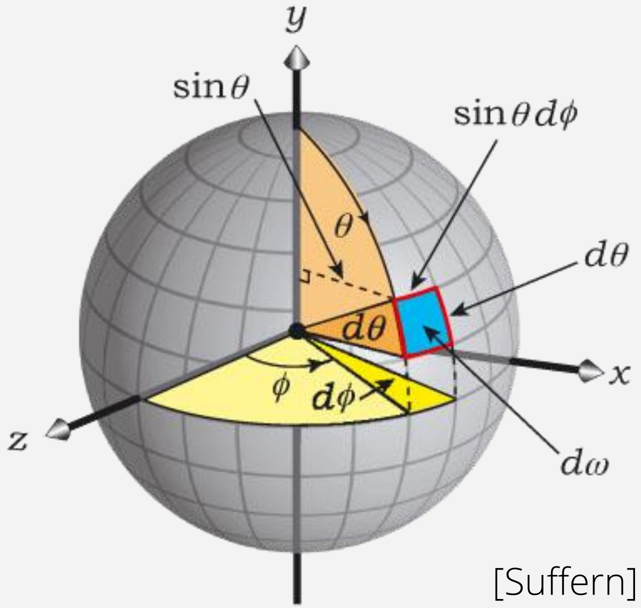
  $$\mathrm{d}A = r\ \mathrm{d}\theta\ r\ \sin\theta\ \mathrm{d}\phi$$ see next slide

- Differential solid angle

  $$\mathrm{d}\omega = \frac{dA}{r^2} = \sin\theta\ \mathrm{d}\theta\ \mathrm{d}\phi$$

- E.g., integral over the hemisphere

  $$\int_{2\pi} f(\boldsymbol{\omega})\ \mathrm{d}\omega = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} f(\theta,\phi) \sin\theta\ \mathrm{d}\theta\ \mathrm{d}\phi$$



[Suffern]

2π indicates the hemisphere above an opaque surface

Integrating Over Solid Angles

- $\mathrm{d}A = \mathrm{d}a_1 \cdot \mathrm{d}a_2$

  $\mathrm{d}a_1 = 2 \cdot r \cdot \sin \frac{\mathrm{d}\theta}{2}$

  $\mathrm{d}a_2 = 2 \cdot r \cdot \sin \frac{\sin\theta \mathrm{d}\phi}{2}$

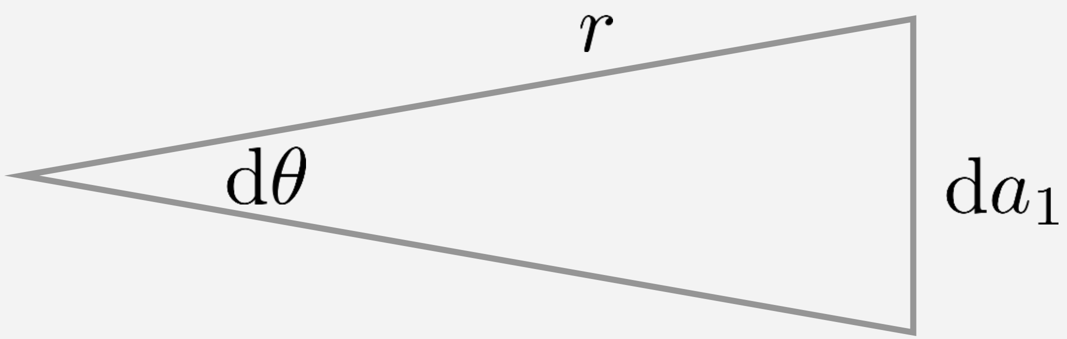
- $\sin(x+h) = \sin(x) + h \cdot \cos(x) + O(h^2)$

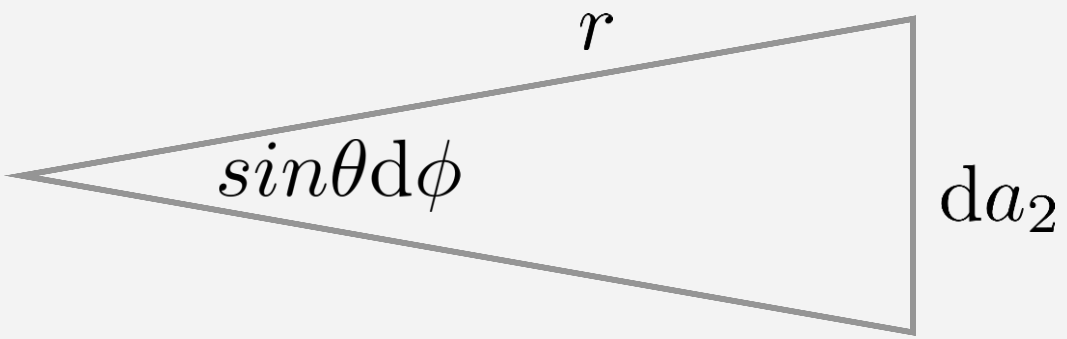
  $x = 0 \rightarrow \sin(h) = h + O(h^2)$

  $\sin(h) \approx h$

- $\mathrm{d}a_1 \approx 2 \cdot r \cdot \frac{\mathrm{d}\theta}{2} \quad \mathrm{d}a_2 \approx 2 \cdot r \cdot \frac{\sin\theta \mathrm{d}\phi}{2}$

  $\mathrm{d}A \approx r^2 \sin\theta \; \mathrm{d}\theta \; \mathrm{d}\phi$





Exemplary Integrals

– Solid angle of a hemisphere

$$\int_{2\pi} 1 \mathrm{d}\omega = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \sin\theta \;\mathrm{d}\theta \;\mathrm{d}\phi = \int_0^{2\pi} [-\cos\theta]_0^{\frac{\pi}{2}} \;\mathrm{d}\phi$$
$$= \int_0^{2\pi} 1 \;\mathrm{d}\phi$$
$$= 2\pi$$

– Solid angle of a sphere

$$\int_{4\pi} 1 \mathrm{d}\omega = \int_0^{2\pi} \int_0^{\pi} \sin\theta \;\mathrm{d}\theta \;\mathrm{d}\phi = \int_0^{2\pi} [-\cos\theta]_0^{\pi} \;\mathrm{d}\phi$$
$$= \int_0^{2\pi} 2 \;\mathrm{d}\phi$$
$$= 4\pi$$

Integrating a Cosine Lobe

- E.g., to determine the irradiance at a point

$$E_i(\boldsymbol{p}) = \int_{2\pi} L_i(\boldsymbol{p}, \boldsymbol{\omega}_i) \cos\theta_i \mathrm{d}\omega_i$$

- General form

$$\int_0^{2\pi} \int_0^{\frac{\pi}{2}} \cos^n\theta \sin\theta \ \mathrm{d}\theta \ \mathrm{d}\phi = \int_0^{2\pi} \left[-\frac{\cos^{n+1}\theta}{n+1} \right]_0^{\frac{\pi}{2}} \ \mathrm{d}\phi = \int_0^{2\pi} \frac{1}{n+1} \ \mathrm{d}\phi = \frac{2\pi}{n+1}$$

- E.g., constant radiance from / into all directions

$$E(\boldsymbol{p}) = \int_{2\pi} L_i(\boldsymbol{p}, \boldsymbol{\omega}_i) \cos\theta_i \mathrm{d}\omega_i = L_i(\boldsymbol{p}) \int_{2\pi} \cos\theta_i \mathrm{d}\omega_i = \pi L_i(\boldsymbol{p})$$

$$B(\boldsymbol{p}) = \int_{2\pi} L_o(\boldsymbol{p}, \boldsymbol{\omega}_o) \cos\theta_o \mathrm{d}\omega_o = L_o(\boldsymbol{p}) \int_{2\pi} \cos\theta_o \mathrm{d}\omega_o = \pi L_o(\boldsymbol{p})$$

- Diffuse reflection $L_o(\boldsymbol{p}) = \frac{B(\boldsymbol{p})}{\pi} = \frac{\rho}{\pi} E(\boldsymbol{p})$

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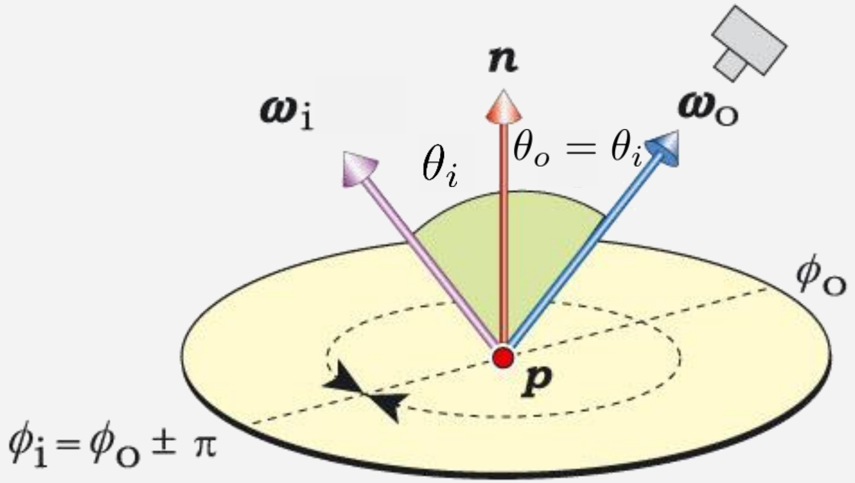
Example – Mirror Reflection

- Each ray of incident flux produces one ray of outgoing flux into a mirrored direction
- Reflectance $0 \leq \rho \leq 1$
- BRDF

$$f_{r,m}(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \rho \frac{1}{\cos\theta_i \sin\theta_i} \delta(\theta_o - \theta_i)\ \delta(\phi_o \pm \pi - \phi_i)$$



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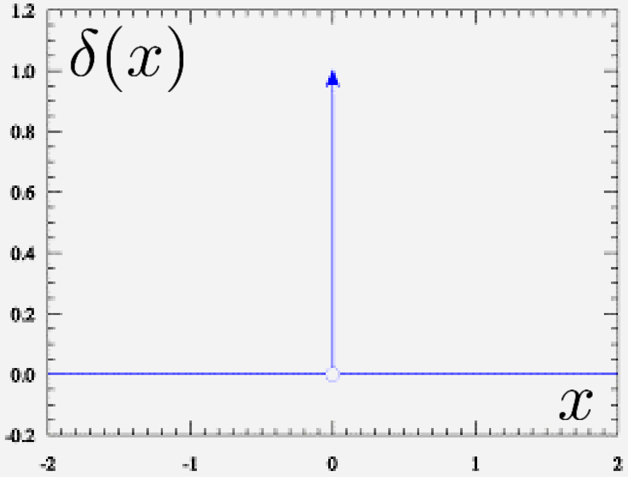
Example – Mirror Reflection

– Delta function

$$x \neq 0 \rightarrow \delta(x) = 0 \qquad \int \delta(x) \mathrm{d}x = 1$$

$$\int f(x)\delta(y-x)\mathrm{d}x = f(y)$$

$$\int f(x)\delta(x-y)\mathrm{d}x = f(y)$$



– Mirrored radiance

$$L_o(\theta_o, \phi_o) = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \rho \frac{1}{\cos\theta_i \sin\theta_i} \delta(\theta_o - \theta_i)\delta(\phi_o \pm \pi - \phi_i) L_i(\theta_i, \phi_i) \cos\theta_i \sin\theta_i \mathrm{d}\theta_i \mathrm{d}\phi_i$$

$$= \rho \int_0^{2\pi} \delta(\phi_o \pm \pi - \phi_i) \int_0^{\frac{\pi}{2}} L_i(\theta_i, \phi_i)\delta(\theta_o - \theta_i)\mathrm{d}\theta_i \mathrm{d}\phi_i$$

$$= \rho \int_0^{2\pi} L_i(\theta_o, \phi_i)\delta(\phi_o \pm \pi - \phi_i)\mathrm{d}\phi_i$$

$$= \rho\, L_i(\theta_o, \phi_o \pm \pi) = \rho\, L_i(\theta_i, \phi_i)$$

BRDF Properties

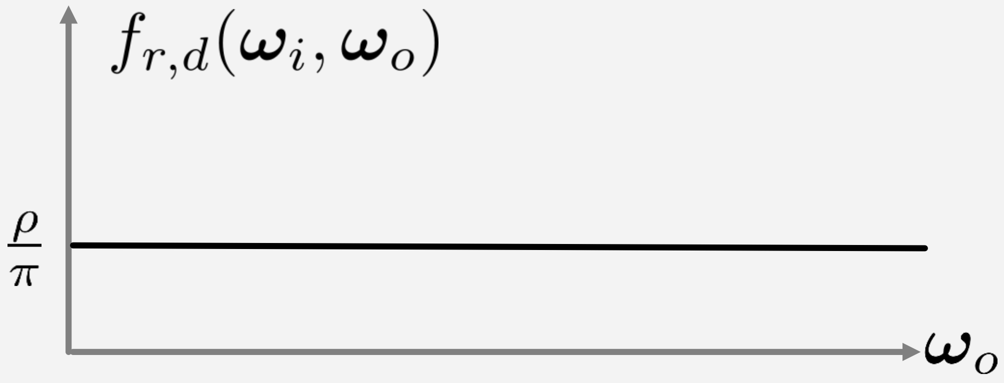
- Values are positive $f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) \geq 0$
  - Incident flux cannot induce negative exitant flux
- Helmholtz reciprocity: $f_r(\boldsymbol{\omega}_o, \boldsymbol{\omega}_i) = f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o)$
  - If incident and exitant flux are reversed, the BRDF value remains the same

BRDF Properties

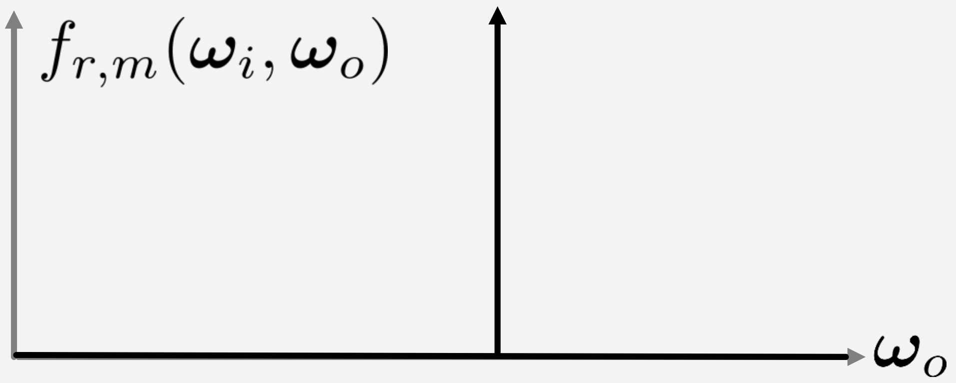
- Values can be arbitrarily large
  - E.g., mirror reflection
  - Ratio of exitant flux to the illumination effect of incident flux

$$f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \frac{\mathrm{d}L_o(\boldsymbol{\omega}_o)}{\mathrm{d}E_i(\boldsymbol{\omega}_i)} = \frac{\mathrm{d}L_o(\boldsymbol{\omega}_o)}{L_i(\boldsymbol{\omega}_i)\cos\theta_i \mathrm{d}\omega_i}$$

Graphs are shown for one specific, fixed ω_i



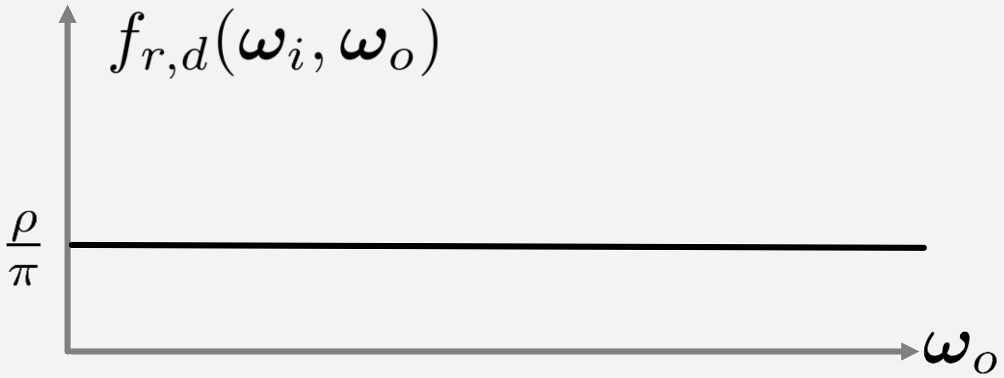
Diffuse reflection



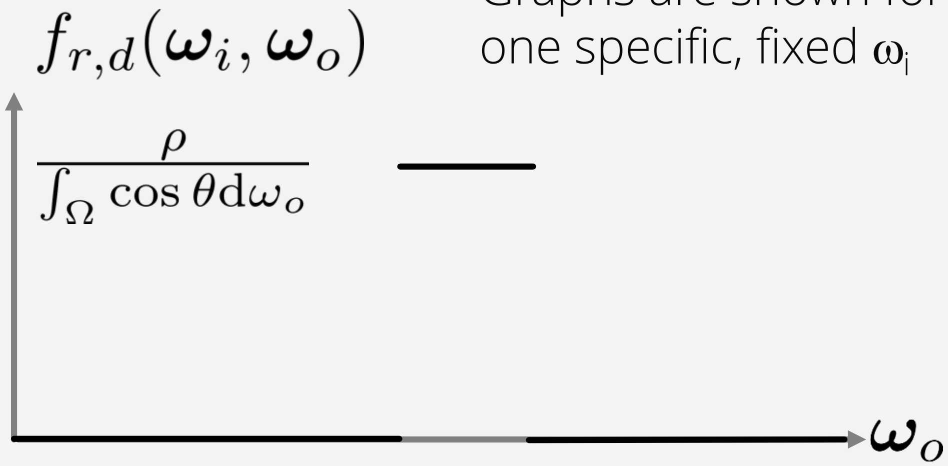
Mirror reflection

BRDF Properties

– Diffuse reflection



Diffuse reflection
into the hemisphere



Graphs are shown for one specific, fixed ω_i

Diffuse reflection
into a solid angle $\Omega < 2\pi$

– If the solid angle of reflected flux gets smaller, the respective BRDF values get larger

Example - Glossy / Specular Reflection

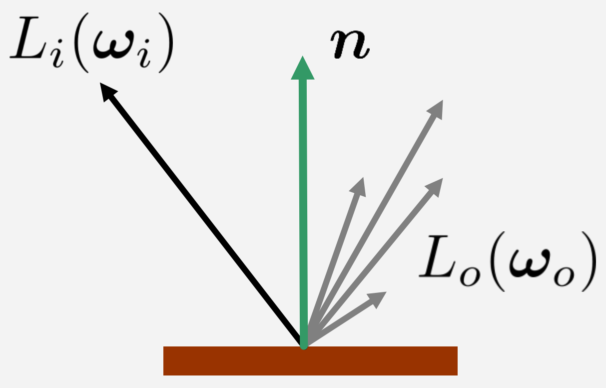
– Incoming flux from direction $\boldsymbol{\omega}_i$ is scattered, but concentrated around the reflection direction $\boldsymbol{\omega}_o$

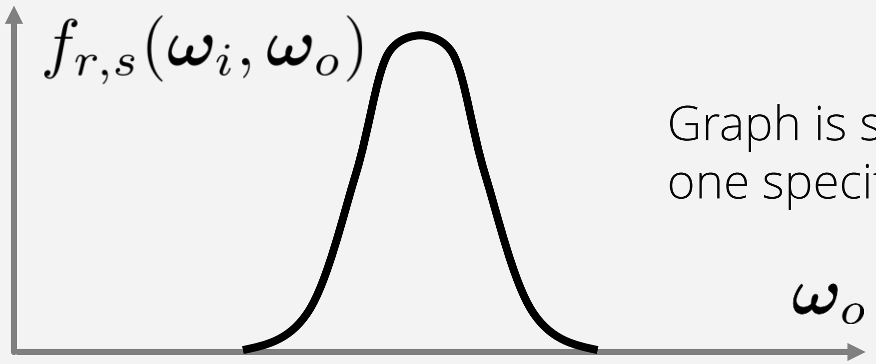
– Reflectance $0 \leq \rho \leq 1$

– BRDF

$$f_{r,s}(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) =$$
$$\rho\ ((2(\boldsymbol{n} \cdot \boldsymbol{\omega}_i) \cdot \boldsymbol{n} - \boldsymbol{\omega}_i) \cdot \boldsymbol{\omega}_o)^e$$

In this term, n, ω_i, ω_o are represented with 3D normalized vectors

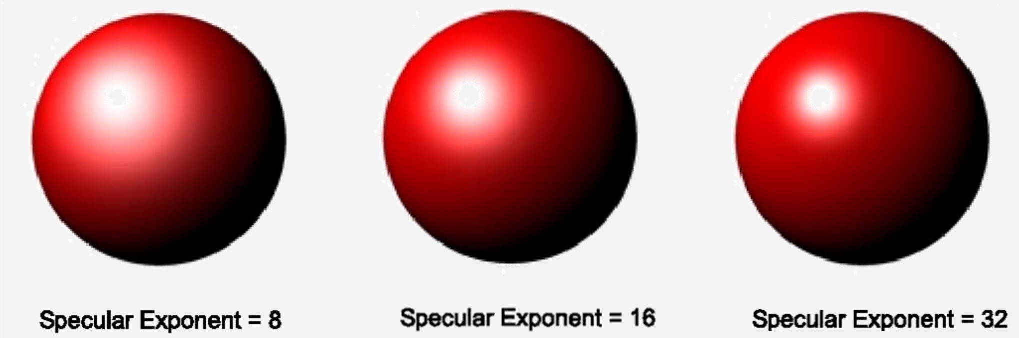




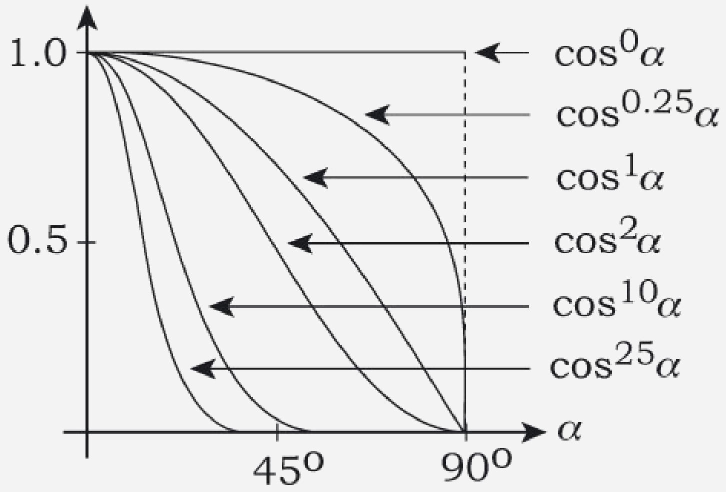
Graph is shown for one specific, fixed ω_i

Example - Glossy / Specular Reflection

- BRDF $f_{r,s}(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \rho \left((2(\boldsymbol{n} \cdot \boldsymbol{\omega}_i) \cdot \boldsymbol{n} - \boldsymbol{\omega}_i) \cdot \boldsymbol{\omega}_o \right)^e$ — n, ω_i, ω_o are represented with 3D normalized vectors
- $\boldsymbol{r}(\boldsymbol{n}, \boldsymbol{\omega}_i) = 2(\boldsymbol{n} \cdot \boldsymbol{\omega}_i) \cdot \boldsymbol{n} - \boldsymbol{\omega}_i$ is the reflection direction of $\boldsymbol{\omega}_i$
- α is the angle between $\boldsymbol{r}(\boldsymbol{n}, \boldsymbol{\omega}_i)$ and $\boldsymbol{\omega}_o$
- $\rho \left((2(\boldsymbol{n} \cdot \boldsymbol{\omega}_i) \cdot \boldsymbol{n} - \boldsymbol{\omega}_i) \cdot \boldsymbol{\omega}_o \right)^e$
  $= \rho \left(\boldsymbol{r}(\boldsymbol{n}, \boldsymbol{\omega}_i) \cdot \boldsymbol{\omega}_o \right)^e = \rho \ \cos^e \alpha$



[Schroecker, Grabner]



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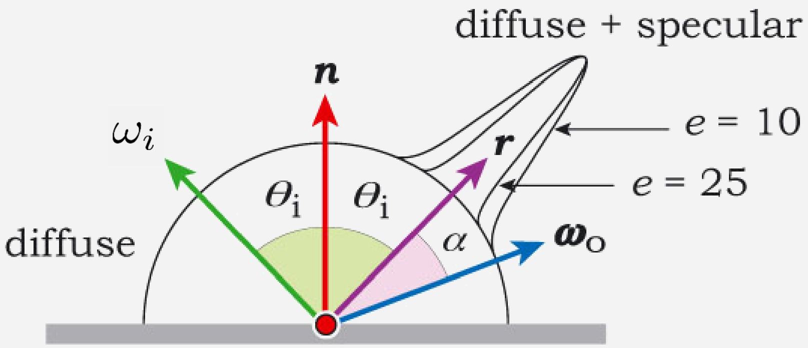
BRDF Properties

- Linearity
  - If a material is defined as a combination of BRDFs, the contributions of the BRDFs are added for the total outgoing radiance



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Exitant radiance for incident radiance from ω_i for a combined material (diffuse + specular, $f_{r,d} + f_{r,s}$)

Example – Mixed Diffuse / Specular Reflection

– Weighted average of diffuse and specular reflection

$$f_{r,ds}(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \alpha \, \frac{\rho_d}{\pi} + \beta \, \rho_s \, ((2(\boldsymbol{n} \cdot \boldsymbol{\omega}_i) \cdot \boldsymbol{n} - \boldsymbol{\omega}_i) \cdot \boldsymbol{\omega}_o)^e$$

– Relation to Phong illumination model

  – Normalized light source direction $\boldsymbol{l}$, viewer $\boldsymbol{v}$, normal $\boldsymbol{n}$

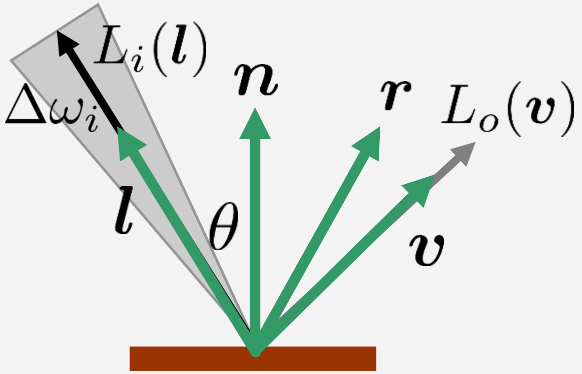
$$L_o^{\mathrm{P}}(\boldsymbol{v}) = \frac{\rho_d}{\pi} \, L_i(\boldsymbol{l}) \, \boldsymbol{n} \cdot \boldsymbol{l} + \rho_s \, L_i(\boldsymbol{l}) \, \boldsymbol{n} \cdot \boldsymbol{l} \, (\boldsymbol{r} \cdot \boldsymbol{v})^e$$

  – Reflection equation

$$L_o^{\mathrm{R}}(\boldsymbol{v}) = \int_{2\pi} f_r(\boldsymbol{\omega}_i, \boldsymbol{v}) L_i(\boldsymbol{\omega}_i) \cos\theta_i \mathrm{d}\omega_i$$

$$\approx f_r(\boldsymbol{l}, \boldsymbol{v}) L_i(\boldsymbol{l}) \, \boldsymbol{n} \cdot \boldsymbol{l} \, \Delta\omega_i$$

$$f_r(\boldsymbol{l}, \boldsymbol{v}) = \frac{\rho_d}{\pi \cdot \Delta\omega} + \frac{\rho_s}{\Delta\omega} (\boldsymbol{r} \cdot \boldsymbol{v})^e \Rightarrow L_o^{\mathrm{P}}(\boldsymbol{v}) = L_o^{\mathrm{R}}(\boldsymbol{v})$$



BRDF Properties - Energy Conservation

Eric Veach: Robust Monte Carlo Methods for Light Transport Simulation, Ph.D. dissertation, Stanford University, 1997.

- Incident radiance / illumination from direction $\boldsymbol{\omega}_i = (\theta_i, \phi_i)$

  $$L_i(\theta,\phi) = \frac{1}{\sin\theta_i \cos\theta_i}\delta(\theta-\theta_i)\delta(\phi-\phi_i)$$

- Irradiance

  $$E = \int\int L_i(\theta,\phi)\ \cos\theta\ \sin\theta\ \mathrm{d}\theta\ \mathrm{d}\phi = 1$$

- Exitant radiance in direction $\boldsymbol{\omega}_o = (\theta_o, \phi_o)$

  $$L_o(\theta_o,\phi_o) = \int\int L_i(\theta,\phi)\ f_r(\theta,\phi,\theta_o,\phi_o)\ \cos\theta\ \sin\theta\ \mathrm{d}\theta\ \mathrm{d}\phi = f_r(\theta_i,\phi_i,\theta_o,\phi_o)$$

- Radiosity

  $$B = \int\int L_o(\theta_o,\phi_0)\ \cos\theta_o \sin\theta_o\, \mathrm{d}\theta_o\, \mathrm{d}\phi_o = \int\int f_r(\theta_i,\phi_i,\theta_o,\phi_o)\ \cos\theta_o\ \sin\theta_o\, \mathrm{d}\theta_o\, \mathrm{d}\phi_o$$

  $$B \leq E \Rightarrow \int\int f_r(\theta_i,\phi_i,\theta_o,\phi_o)\ \cos\theta_o\ \sin\theta_o\ \mathrm{d}\theta_o\ \mathrm{d}\phi_o \leq 1$$

  $$\forall\boldsymbol{\omega}_i : \int_{2\pi} f_r(\boldsymbol{\omega}_i,\boldsymbol{\omega}_o)\ \cos\theta_o\ \mathrm{d}\omega_o \leq 1$$

BRDF Summary - Properties

- Definition: $f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \frac{\mathrm{d}L_o(\boldsymbol{\omega}_o)}{\mathrm{d}E_i(\boldsymbol{\omega}_i)} = \frac{\mathrm{d}L_o(\boldsymbol{\omega}_o)}{L_i(\boldsymbol{\omega}_i) \cdot \cos\theta_i \cdot \mathrm{d}\omega_i}$
- Positive: $f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) \geq 0$
- Helmholtz reciprocity: $f_r(\boldsymbol{\omega}_o, \boldsymbol{\omega}_i) = f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o)$
  - Incident and exitant radiance can be reversed
- Energy conservation: $\forall \boldsymbol{\omega}_i : \int_{2\pi} f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) \cos\theta_o \mathrm{d}\boldsymbol{\omega}_o \leq 1$
- Linearity
  - If a material is defined as a sum of BRDFs, the contributions of the BRDFs are added for the total outgoing radiance
    $\int (f_{r,1} + f_{r,2}) L_i \cos\theta_i \mathrm{d}\omega_i = \int f_{r,1} L_i \cos\theta_i \mathrm{d}\omega_i + \int f_{r,2} L_i \cos\theta_i \mathrm{d}\omega_i$

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BRDF Summary – Exemplary Materials

- Diffuse

$$f_{r,d}(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \frac{\rho}{\pi}$$

- Mirror

$$f_{r,m}(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \rho \; \frac{1}{\cos\theta_i \sin\theta_i} \; \delta(\theta_o - \theta_i) \; \delta(\phi_o \pm \pi - \phi_i)$$

- Specular

$$f_{r,s}(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \rho \; ((2(\boldsymbol{n} \cdot \boldsymbol{\omega}_i) \cdot \boldsymbol{n} - \omega_i) \cdot \boldsymbol{\omega}_o)^e$$

n, ω_i, ω_o are represented with 3D normalized vectors

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Advanced Computer Graphics
Materials 3

Matthias Teschner

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Outline

- Context
- Bidirectional Reflectance Distribution Function BRDF
  - Definition
  - Application
  - Exemplary materials
  - Properties
- Reflectance
- Material modeling
- Transparent materials

Reflectance

- Ratio of outgoing flux to incident flux
  - Surface property, used in BRDFs
- Incident flux $\mathrm{d}\Phi_i = \mathrm{d}A \int_{\Omega_i} L_i(\boldsymbol{\omega}_i) \cos\theta_i \mathrm{d}\omega_i \quad (E = \frac{\mathrm{d}\Phi}{\mathrm{d}A} = \int_{\Omega_i} L_i \cos\theta_i \mathrm{d}\omega_i)$
- Outgoing flux $\mathrm{d}\Phi_o = \mathrm{d}A \int_{\Omega_o} L_o(\boldsymbol{\omega}_o) \cos\theta_o \mathrm{d}\omega_o$

$$\mathrm{d}\Phi_o = \mathrm{d}A \int_{\Omega_o} \int_{\Omega_i} f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) L_i(\boldsymbol{\omega}_i) \cos\theta_i \cos\theta_o \mathrm{d}\omega_i \mathrm{d}\omega_o$$

- Reflectance

$$\rho(\Omega_i, \Omega_o) = \frac{\mathrm{d}\Phi_o}{\mathrm{d}\Phi_i} = \frac{\mathrm{d}A \int_{\Omega_o} L_o(\boldsymbol{\omega}_o) \cos\theta_o \mathrm{d}\omega_o}{\mathrm{d}A \int_{\Omega_i} L_i(\boldsymbol{\omega}_i) \cos\theta_i \mathrm{d}\omega_i} = \frac{\mathrm{d}A \int_{\Omega_o} \int_{\Omega_i} f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) L_i(\boldsymbol{\omega}_i) \cos\theta_i \cos\theta_o \mathrm{d}\omega_i \mathrm{d}\omega_o}{\mathrm{d}A \int_{\Omega_i} L_i(\boldsymbol{\omega}_i) \cos\theta_i \mathrm{d}\omega_i}$$

- Various parameterizations of the reflectance can be considered for various configurations of Ω_i, Ω_o

Bihemispherical Reflectance

– Diffuse material

$$\rho(2\pi, 2\pi) = \frac{\mathrm{d}\Phi_o}{\mathrm{d}\Phi_i} = \frac{\mathrm{d}A \int_{2\pi} L_o(\boldsymbol{\omega}_o) \cos\theta_o \mathrm{d}\omega_o}{\mathrm{d}A \int_{2\pi} L_i(\boldsymbol{\omega}_i) \cos\theta_i \mathrm{d}\omega_i}$$

$$= \frac{\mathrm{d}L_o \int_{2\pi} \cos\theta_o \mathrm{d}\omega_o}{\mathrm{d}E(\boldsymbol{\omega_i})}$$

Exitant radiance is equal in all direction

$$= \pi \frac{\mathrm{d}L_o}{\mathrm{d}E(\omega_{\mathbf{i}})}$$

$$= \pi f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o)$$

BRDF definition

– The reflectance in the BRDF for diffuse material is a bihemispherical reflectance $f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \frac{\rho(2\pi, 2\pi)}{\pi}$

– The ratio of exitant and incident flux is independent from the directions of incident and exitant flux

Spectral Bihemispherical Reflectance

- Spectral reflectance is wavelength dependent

$$\rho_\lambda(\Omega_i, \Omega_o) = \frac{\mathrm{d}\Phi_{o,\lambda}}{\mathrm{d}\Phi_{i,\lambda}}$$

- Diffuse spectral reflectance
  (spectrum represented with RGB values)

$$\rho_{\mathrm{red}} = \pi \frac{\mathrm{d}L_{o,\mathrm{red}}}{\mathrm{d}E_{\mathrm{red}}} \quad \rho_{\mathrm{green}} = \pi \frac{\mathrm{d}L_{o,\mathrm{green}}}{\mathrm{d}E_{\mathrm{green}}} \quad \rho_{\mathrm{blue}} = \pi \frac{\mathrm{d}L_{o,\mathrm{blue}}}{\mathrm{d}E_{\mathrm{blue}}}$$

RGB surface color

- Diffuse spectral BRDF

$$f_{r,d,\mathrm{red}}(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \frac{\rho_{\mathrm{red}}}{\pi} \quad f_{r,d,\mathrm{green}}(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \frac{\rho_{\mathrm{green}}}{\pi} \quad f_{r,d,\mathrm{blue}}(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \frac{\rho_{\mathrm{blue}}}{\pi}$$

Bidirectional Reflectance

- How much of the flux from an incident direction is reflected into an exitant direction

$$L_i(\boldsymbol{\omega}_i) = \frac{\mathrm{d}^2\Phi_i}{\mathrm{d}A \ \cos\theta_i \ \mathrm{d}\omega_i} \Rightarrow \mathrm{d}^2\Phi_i = \mathrm{d}A \ L_i(\boldsymbol{\omega}_i) \ \cos\theta_i \ \mathrm{d}\omega_i$$

$$\mathrm{d}L_o(\boldsymbol{\omega}_o) = f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) \ L_i(\boldsymbol{\omega}_i) \cos\theta_i \mathrm{d}\omega_i = \frac{\mathrm{d}^3\Phi_o}{\mathrm{d}A \ \cos\theta_o \ \mathrm{d}\omega_o}$$

$$\Rightarrow \mathrm{d}^3\Phi_o = f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) \ L_i(\boldsymbol{\omega}_i) \cos\theta_i \mathrm{d}\omega_i \mathrm{d}A \ \cos\theta_o \ \mathrm{d}\omega_o$$

$$\mathrm{d}\rho(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \frac{\mathrm{d}^3\Phi_o}{\mathrm{d}^2\Phi_i} = \frac{f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) \ L_i(\boldsymbol{\omega}_i) \cos\theta_i \mathrm{d}\omega_i \mathrm{d}A \ \cos\theta_o \ \mathrm{d}\omega_o}{\mathrm{d}A \ L_i(\boldsymbol{\omega}_i) \ \cos\theta_i \ \mathrm{d}\omega_i}$$

$$\mathrm{d}\rho(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) \cos\theta_o \mathrm{d}\omega_o$$

- Dependent on incident and exitant flux directions

Directional-Hemispherical Reflectance

$$\mathrm{d}\rho(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) \cos\theta_o \mathrm{d}\omega_o$$

$$\rho(\boldsymbol{\omega}_i, 2\pi) = \int_{2\pi} f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) \cos\theta_o \mathrm{d}\omega_o$$

- The term $0 \leq \int_{2\pi} f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) \cos\theta_o \mathrm{d}\omega_o \leq 1$ represents the ratio of exitant flux into the hemisphere and incident flux from direction ω_i
- Another intuition for the energy conservation constraint of a BRDF

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Material Reflectances

– Diffuse, mirror, specular

$$f_{r,d}(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \frac{\rho_d}{\pi}$$

$$f_{r,m}(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \rho_m \; \frac{1}{\cos\theta_i \sin\theta_i} \; \delta(\theta_o - \theta_i) \; \delta(\phi_o \pm \pi - \phi_i)$$

$$f_{r,s}(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \rho_s \; ((2(\boldsymbol{n} \cdot \boldsymbol{\omega}_i) \cdot \boldsymbol{n} - \boldsymbol{\omega}_i) \cdot \boldsymbol{\omega}_o)^e$$

n, ω_i, ω_o are represented with 3D normalized vectors

– In a diffuse BRDF, ρ_d accounts for diffuse reflectance, i.e. the surface color. In glossy, specular, mirror BRDFs, ρ_m, ρ_s is typically (1,1,1), i.e. white. The color of specular highlights converges towards the color of the light source.

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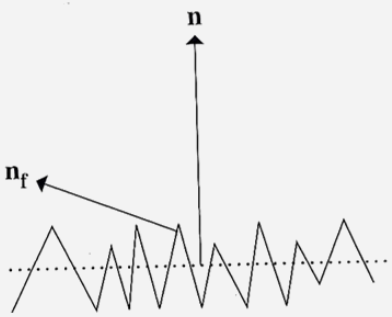
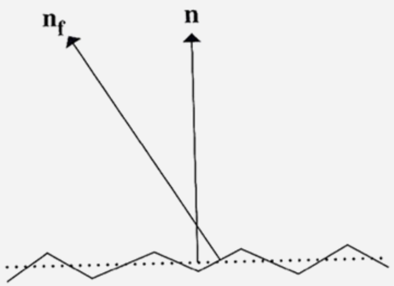
Microfacet Model

- Light scattering at a surface is determined by the distribution of microfacets
- Facets reflect perfectly specular or perfectly diffuse
- Distribution of microfacet normals governs material



Rough surface
Large variation of facet normals



Smooth surface
Small variation of facet normals

[Pharr, Humphreys]

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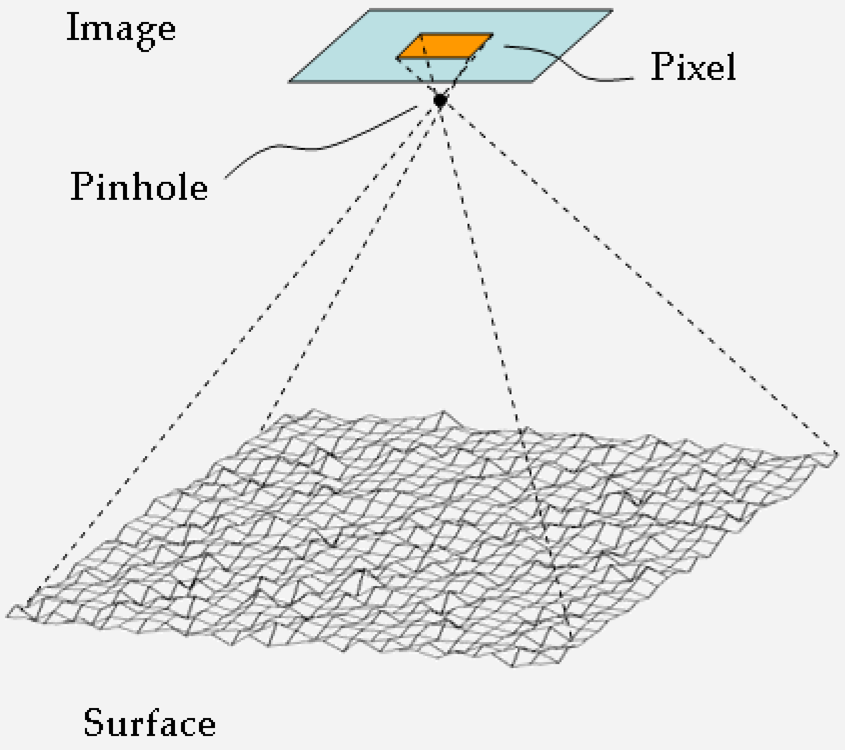
Diffuse Reflecting Material

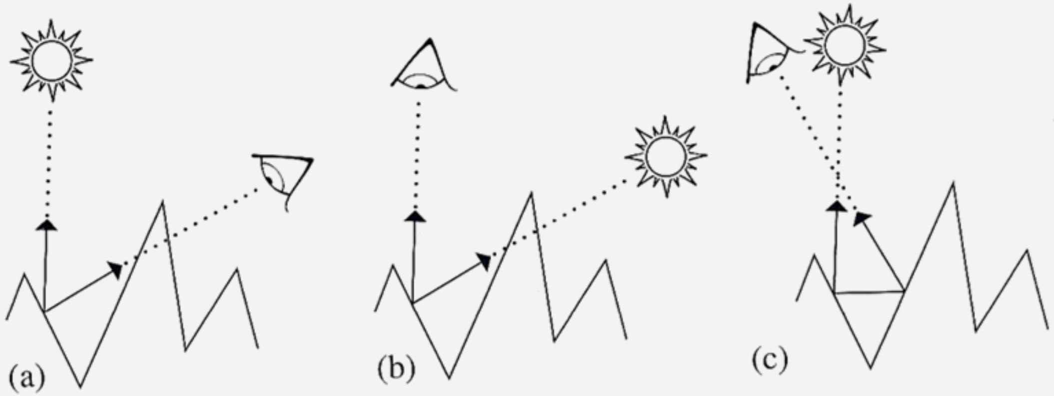
- Facets are perfectly diffuse
- Flux into a detector element is an aggregate value over many facets
- Due to local facet effects, the radiance is not perfectly Lambertian, but potentially view-dependent



[Wikipedia: Oren-Nayar reflectance model]

Local Facet Effects

- Masking
  - Microfacet is not visible to the viewer
- Shadowing
  - Microfacet is not illuminated
- Interreflection
  - Microfacets illuminate each other (additional illumination)



masking shadowing interreflection

[Pharr, Humphreys]

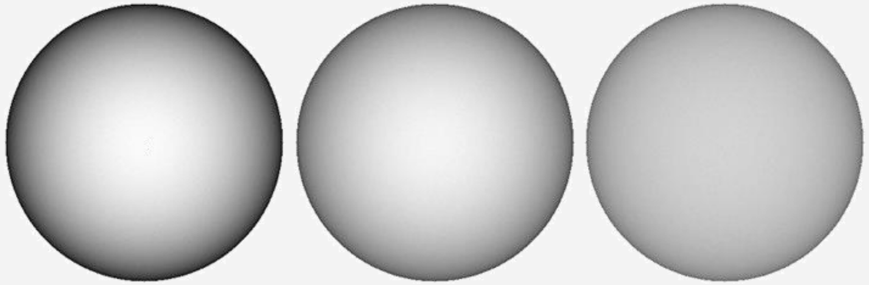
Oren-Nayar Diffuse Material

- Models rough opaque diffuse surfaces
- Facets are Lambertian, symmetric V-shaped grooves
- Gaussian distribution models the facet normals
- Parameter $0 \leq \sigma^2 \leq 1$ gives the variance of the angle between surface normal and facet normal
- viewer direction $\boldsymbol{\omega}_o = (\theta_o, \phi_o)$, light direction $\boldsymbol{\omega}_i = (\theta_i, \phi_i)$
- BRDF $f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \frac{\rho_d}{\pi}(A + B \max(0, \cos(\phi_i - \phi_o)) \sin\alpha \tan\beta)$

$$A = 1 - \frac{\sigma^2}{2(\sigma^2 + 0.33)} \quad B = \frac{0.45\sigma^2}{\sigma^2 + 0.09} \quad \alpha = \max(\theta_i, \theta_o) \quad \beta = \min(\theta_i, \theta_o)$$

Oren-Nayar Diffuse Reflectance

– Results

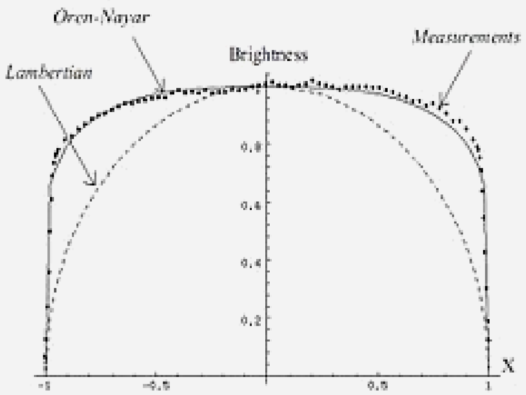


$\sigma = 0$ $\sigma = 0.1$ $\sigma = 0.3$

[Wikipedia: Oren-Nayar reflectance model]

– Comparisons [Oren,Nayar, ACM SIGGRAPH 1994]

Real Image Lambertian Model Oren-Nayar Model



Oren-Nayar vs. Lambertian

- Is brighter for large angles between viewer and surface normal compared to a Lambertian surface
- Gets brighter if the angle between viewer and light gets smaller
- Converges to a Lambertian surface

$$f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \frac{\rho_d}{\pi}(A + B \max(0, \cos(\phi_i - \phi_o)) \sin\alpha \tan\beta)$$

$$A = 1 - \frac{\sigma^2}{2(\sigma^2 + 0.33)} \qquad B = \frac{0.45\sigma^2}{\sigma^2 + 0.09}$$

- If all facets have the same normal, then $\sigma = 0$, $A = 1$, $B = 0$: $f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \frac{\rho_d}{\pi}$

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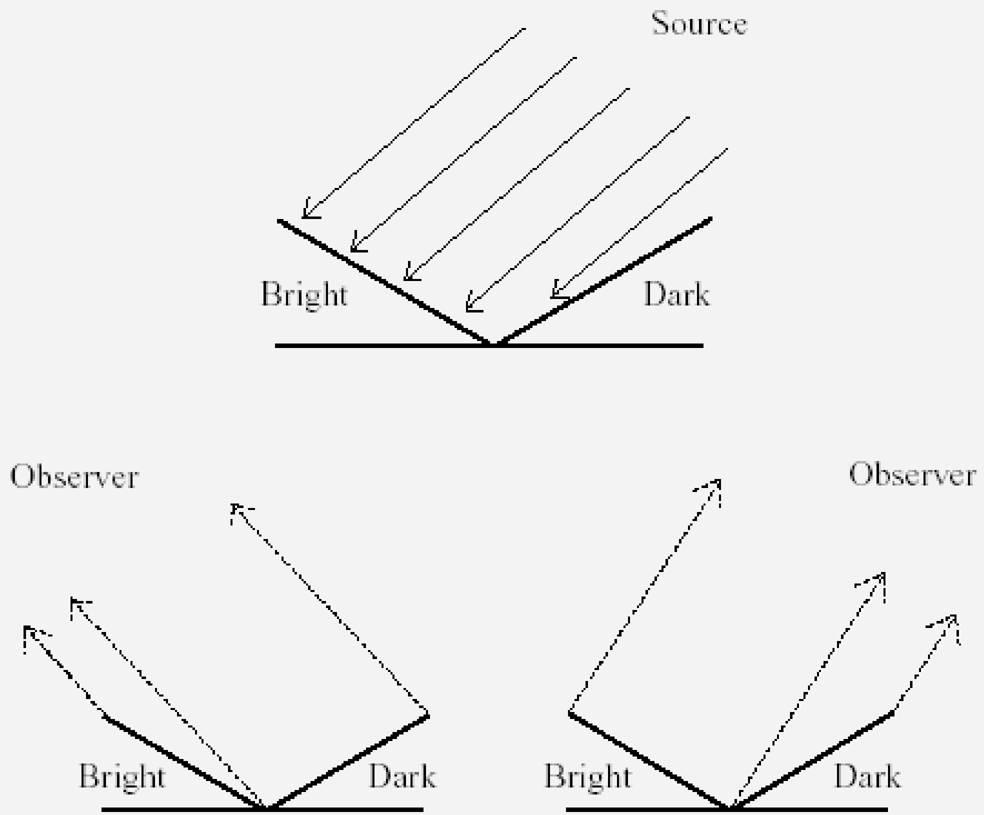
Oren-Nayar - View Dependence

- Different combinations of projected areas of dark and bright facets for different viewer directions
- Maximal brightness, if viewing direction corresponds to light direction



[Srinivasa Narasimhan http://www.cs.cmu.edu/afs/cs/academic/class/16823-f06/]

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Torrance-Sparrow Specular Material

- Models rough opaque specular surfaces
- Facets are perfect mirrors, symmetric and V-shaped
- Beckmann distribution models the facet normals
- Similar models: Cook-Torrance, Blinn
- Viewer direction $\boldsymbol{\omega}_o = (\theta_o, \phi_o)$
- Light source $\boldsymbol{\omega}_i = (\theta_i, \phi_i)$
- Halfway direction $\boldsymbol{\omega}_h(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o)$
- BRDF $f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \frac{D(\boldsymbol{\omega}_h) \;\; G(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) \;\; F(\boldsymbol{\omega}_o)}{\pi \;\; \cos\theta_i \;\; \cos\theta_o}$

D - Beckmann distribution
G - geometric term for shadowing
F - Fresnel term

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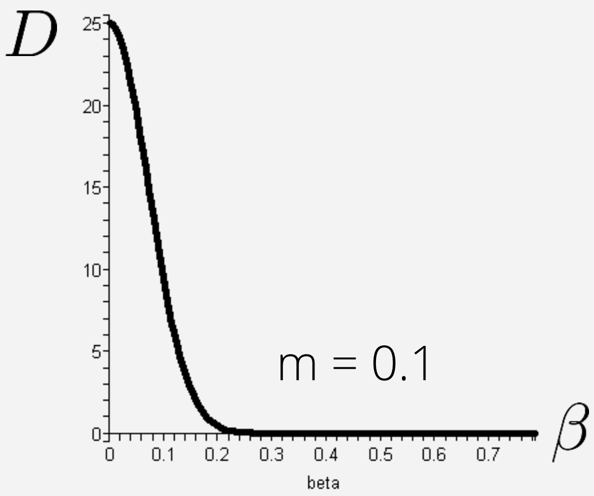
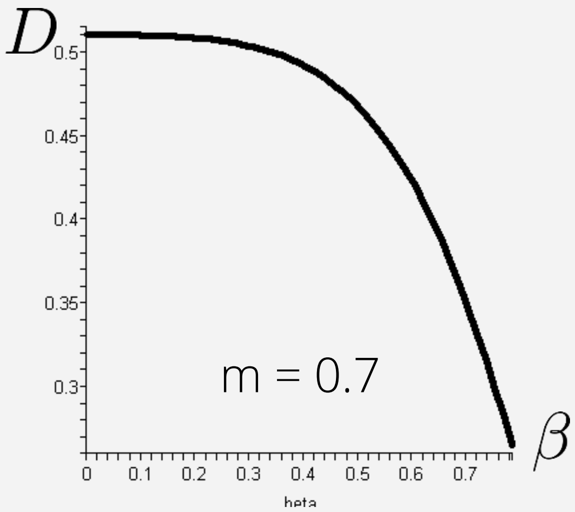
Beckmann Distribution

- Describes the roughness of the surface
- *m* describes the roughness
- β is the angle between halfway vector and surface normal
- $D(\boldsymbol{\omega}_h) = \dfrac{e^{-\frac{\tan^2\beta}{m^2}}}{4m^2\cos^4\beta}$





Geometric Term / Fresnel Term

- Geometric term
  - Accounts for self-shadowing effects of microfacets
- Fresnel term
  - Accounts for a varying absorbance / reflection ratio depending on the angle of incident flux (direction dependent reflectance)
  - Many surfaces reflect more strongly if illumination is nearly parallel to the surface

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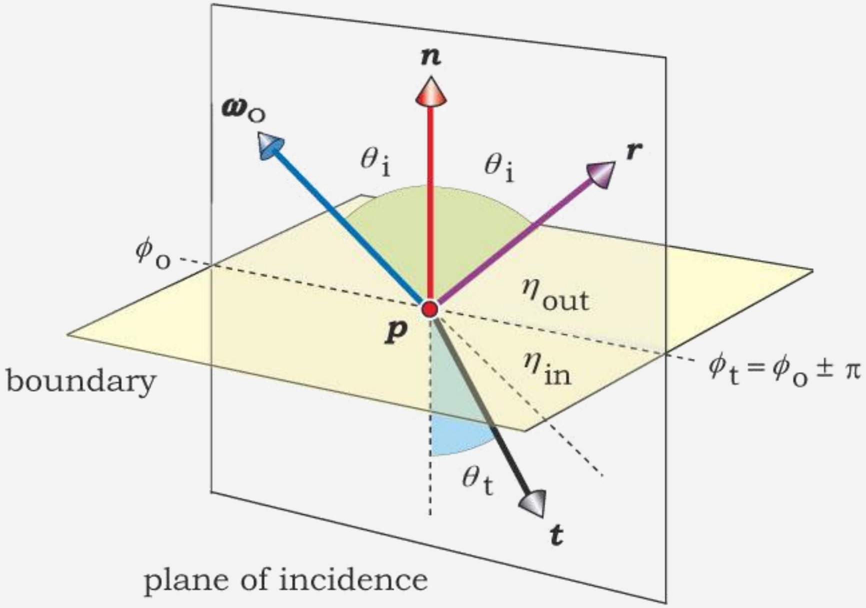
Concept

- Light is reflected and refracted at the interface between two transparent media
- Incident, reflected, and transmitted rays are in the same plane
- Direction $\boldsymbol{t}$ is determined by ω_o and by the indices of refraction η_{out} and η_{in}



[Suffern]

Snell's Law

- Materials are characterized by a refractive index
  - Ratio of the speed of light in vacuum and in the considered medium
  - Vacuum 1.0, air 1.0003, ice 1.31, water 1.33, glass 1.5
- Snell's law / law of refraction
  - $\frac{\sin\theta_i}{\sin\theta_t} = \frac{\eta_{in}}{\eta_{out}} = \eta$ Relative refraction index
- Transmission direction
  - $\boldsymbol{t} = \frac{1}{\eta}\boldsymbol{\omega}_o - (\cos\theta_t - \frac{1}{\eta}\cos\theta_i)\boldsymbol{n}$
  - $\cos\theta_t = (1 - \frac{1}{\eta^2}(1 - \cos^2\theta_i))^{\frac{1}{2}}$
  - All vectors should be normalized, then *t* is also normalized

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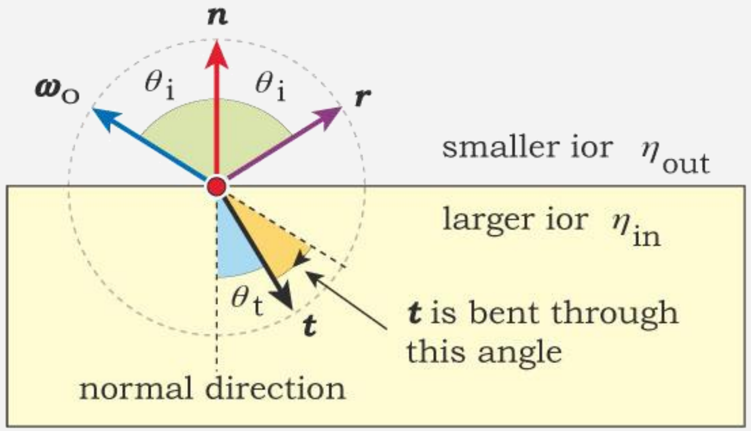
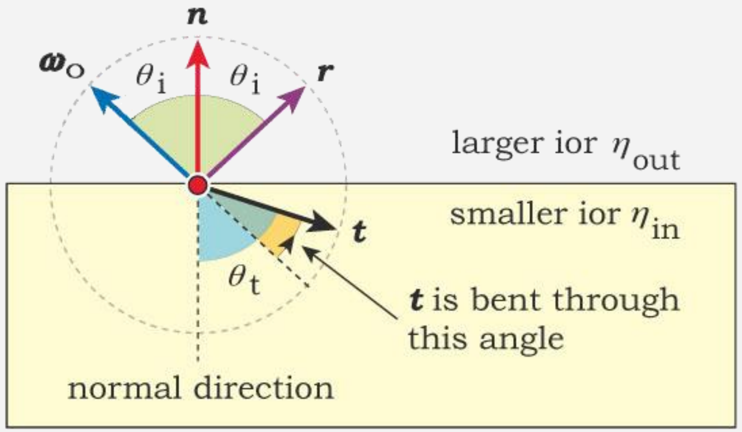
Snell's Law

– For $\eta>1$, transmitted rays are bent towards the normal



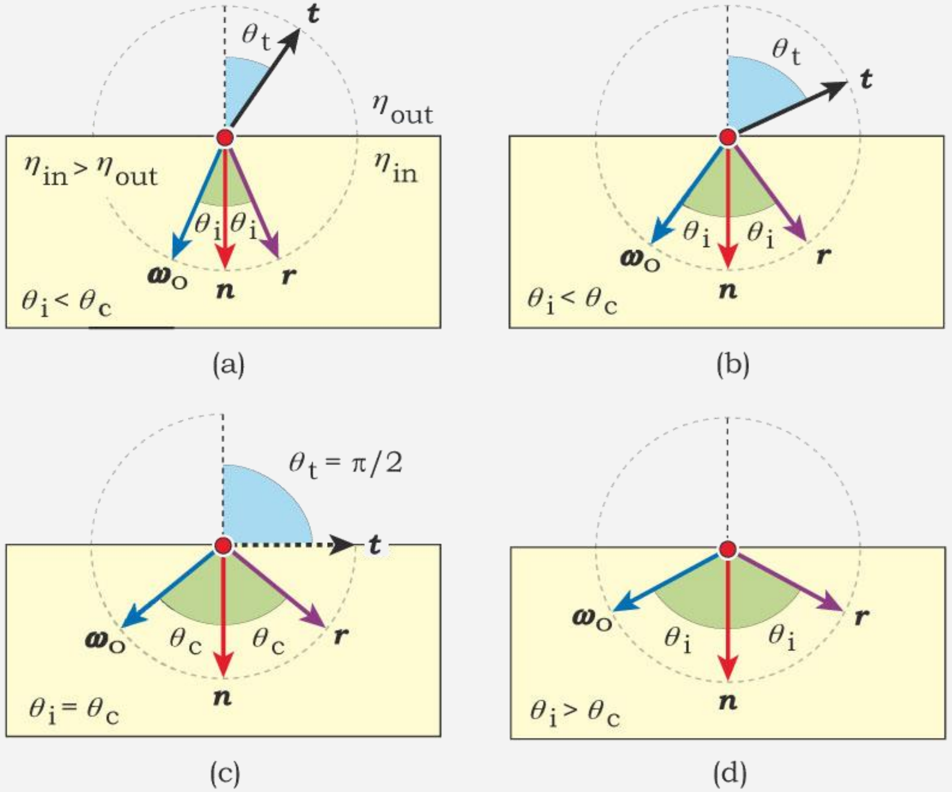
– For $\eta<1$, transmitted rays are bent away from the normal



[Suffern]

Total Internal Reflection

- For η<1 and θ_i>θ_c (critical angle), total internal reflection occurs
- Transmitted ray is bent towards the interface until the incident light ray reaches the critical angle
- For θ_i=θ_c: $1 - \frac{1}{\eta^2}(1 - \cos^2\theta_i) = 0$
- For θ_i>θ_c: $1 - \frac{1}{\eta^2}(1 - \cos^2\theta_i) < 0$



[Suffern]

Reflectance Equation

- Incident, BRDF weighted radiance is integrated over the hemisphere to compute the outgoing radiance

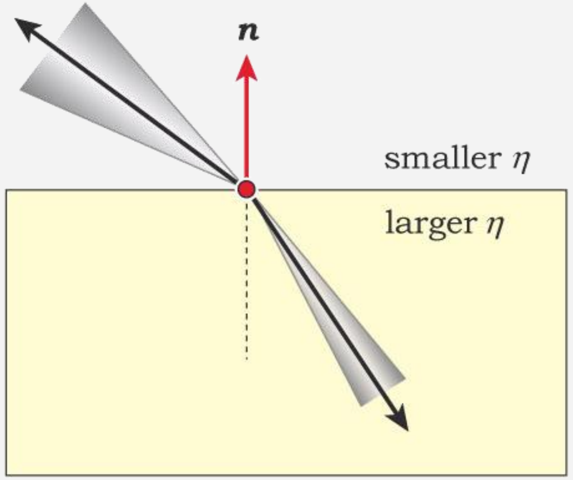
  $$L_o(\boldsymbol{\omega}_o) = \int_{2\pi} f_r(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) L_i(\boldsymbol{\omega}_i) \cos\theta_i \mathrm{d}\omega_i$$

- Can also consider transmitted light
- BTDF - bidirectional transmittance distribution function
  - $f_t(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o)$ with ω_i and ω_o in opposite hemispheres
- BRDF and BTDF can be combined to a BSDF - bidirectional scattering distribution function

  $$L_o(\boldsymbol{\omega}_o) = \int_{4\pi} f(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) L_i(\boldsymbol{\omega}_i) |\cos\theta_i| \mathrm{d}\omega_i$$

Specular Transmission

- Snell's law gives the direction of a refracted ray and its radiance change
- Motivation
  - Solid angle of a differential cone of incident radiance changes due to refraction
- $L_t = \rho_t \frac{\eta_t^2}{\eta_i^2} L_i$ with $0 \leq \rho_t \leq 1$ being the transmission coefficient
- Resulting BTDF: $f_t(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \rho_t \frac{\eta_t^2}{\eta_i^2} \frac{\delta(\boldsymbol{\omega}_i - \boldsymbol{t}(\boldsymbol{n}, \boldsymbol{\omega}_o))}{\sin\theta_i |\cos\theta_i|}$



[Suffern]

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Specular Transmission

- Ray and radiance-transfer direction
- At point *a*, radiance transfer is from inside to outside

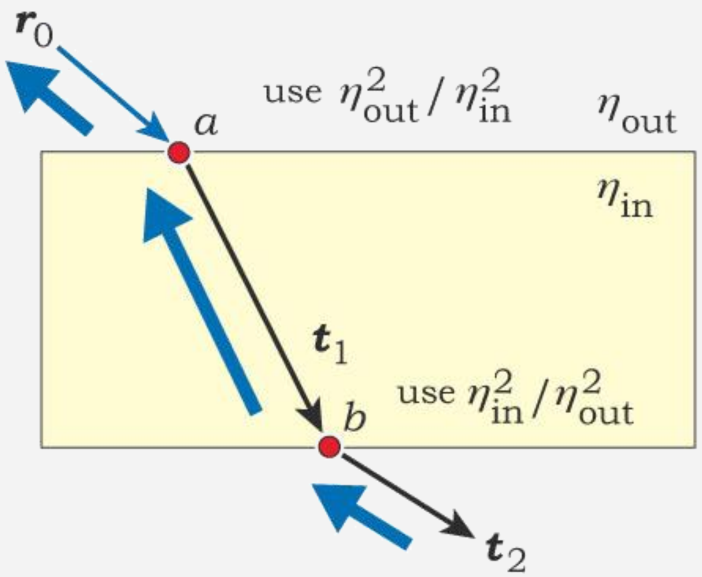
$$\eta_i = \eta_{in} \quad \eta_t = \eta_{out}$$

- At point *b*, radiance transfer is from outside to inside

$$\eta_i = \eta_{out} \quad \eta_t = \eta_{in}$$

- Radiance changes cancel when a ray enters and leaves a medium

$$L_t = \rho_t \frac{\eta_t^2}{\eta_i^2} L_i$$



[Suffern]

Reflectance and Transmittance

- Fresnel equations describe which fractions of incident light are reflected and transmitted
- Fractions depend on the refraction indices and on the incident light direction
- Fresnel reflectance $F_r = \frac{1}{2}\left(\left(\frac{\eta\cos\theta_i - \cos\theta_t}{\eta\cos\theta_i + \cos\theta_t}\right)^2 + \left(\frac{\cos\theta_i - \eta\cos\theta_t}{\cos\theta_i + \eta\cos\theta_t}\right)^2\right)$
- Fresnel transmittance $F_t = 1 - F_r$

Reflectance and Transmittance

- General form

$$F_r = \frac{1}{2}\left(\left(\frac{\eta\cos\theta_i - \cos\theta_t}{\eta\cos\theta_i + \cos\theta_t}\right)^2 + \left(\frac{\cos\theta_i - \eta\cos\theta_t}{\cos\theta_i + \eta\cos\theta_t}\right)^2\right)$$

- Normal incidence $\theta_i = \theta_t = 0$

$$F_r = \frac{1}{2}\left(\left(\frac{\eta-1}{\eta+1}\right)^2 + \left(\frac{1-\eta}{1+\eta}\right)^2\right)$$

  - E.g., $F_r = 0.04$ at the interface of air and glass with $\eta = 1.5$
- Grazing incidence $\theta_i = \frac{\pi}{2}$

$$F_r = 1$$

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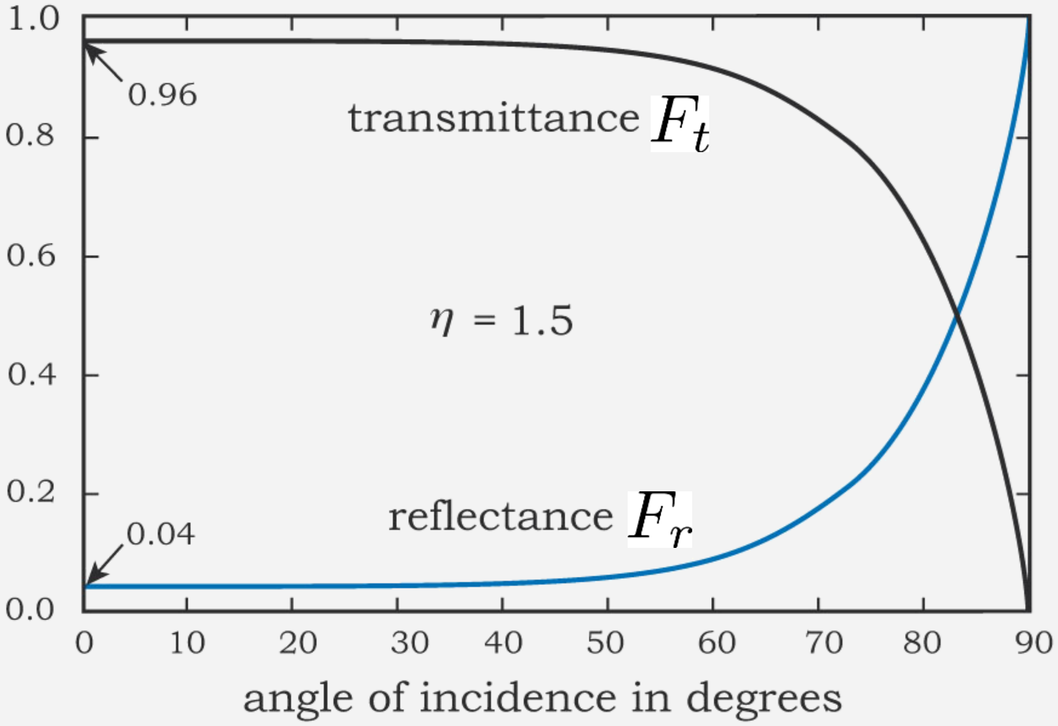
Reflectance and Transmittance

– Reflectance and transmittance of glass for varying incident angles



values are approximately constant up to 50 degrees

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Light Attenuation

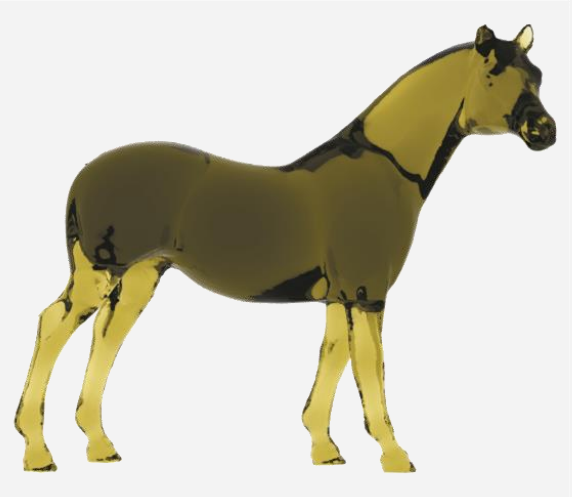
- Transparent materials attenuate light
  - Light is scattered / absorbed along the ray
  - Radiance decreases exponentially with distance
  - Beer-Lambert law
  - Transparent materials are examples of participating media

Light Attenuation

- $L_a = c_f^d L_b$
- d is the distance inside a medium
- L_a is exitant radiance, L_b is incident radiance
- c describes the attenuation (wavelength-dependent)
  - c = (1,1,1) - no attenuation
  - c = (1,1,0.9) - blue is attenuated, resulting in a yellow appearance

Light Attenuation

– Examples



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