Surface Simplification Using Quadric Error Metrics

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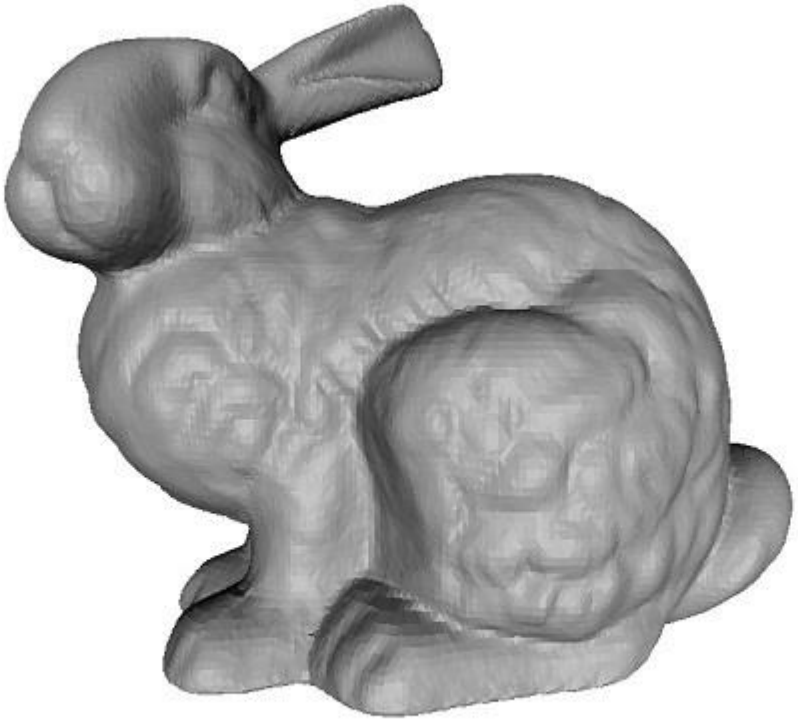
Content

Surface Simplification



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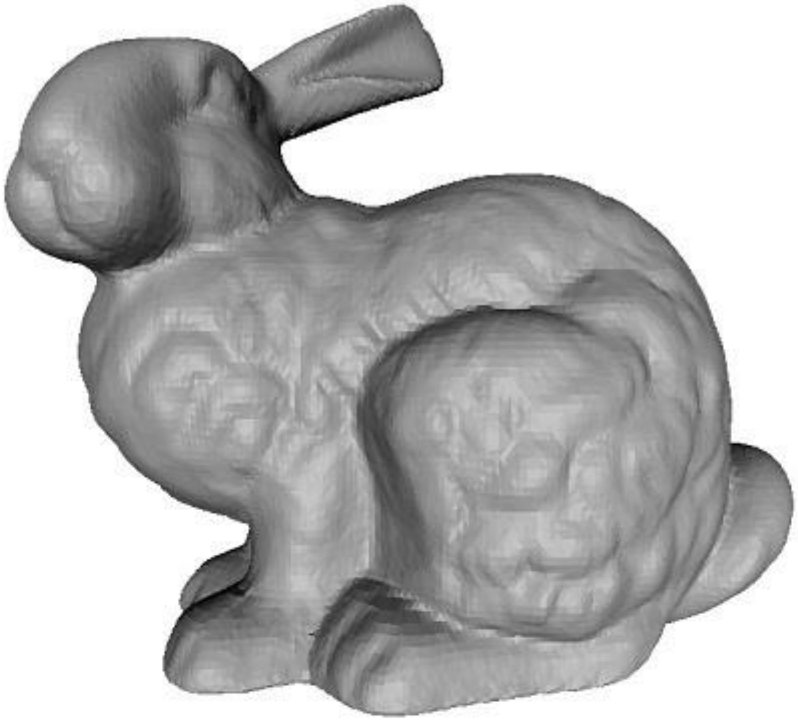
Surface Simplification



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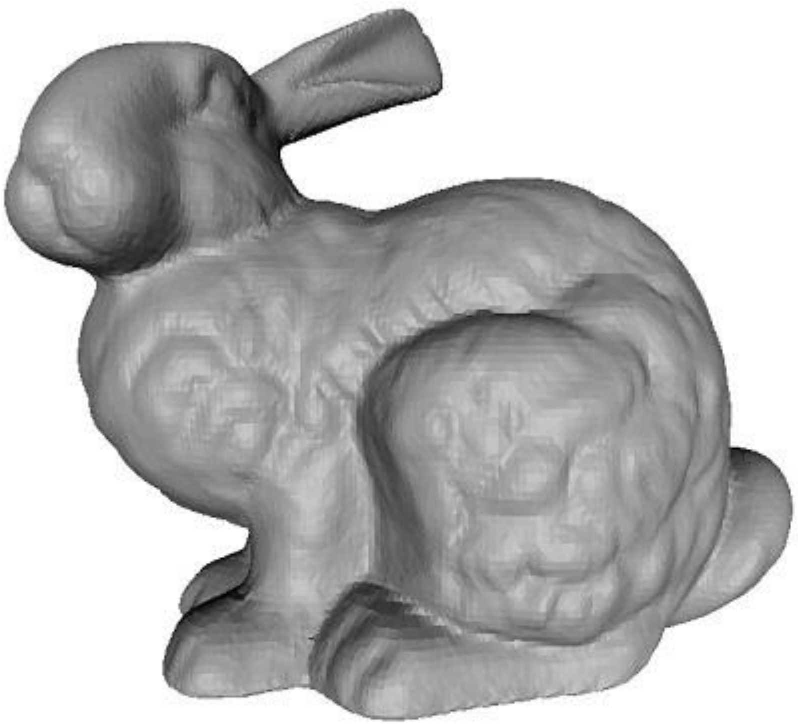
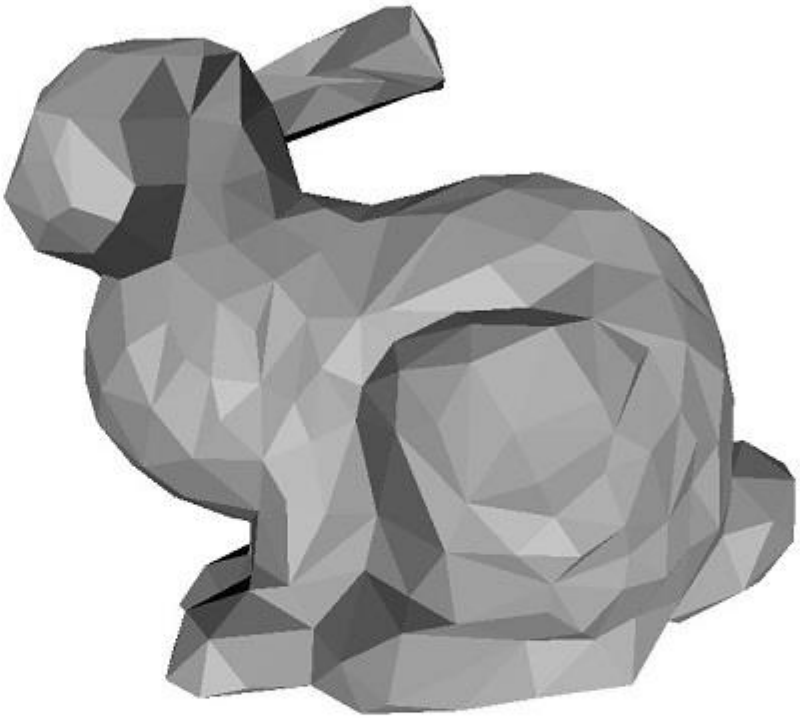
Surface Simplification





[4]

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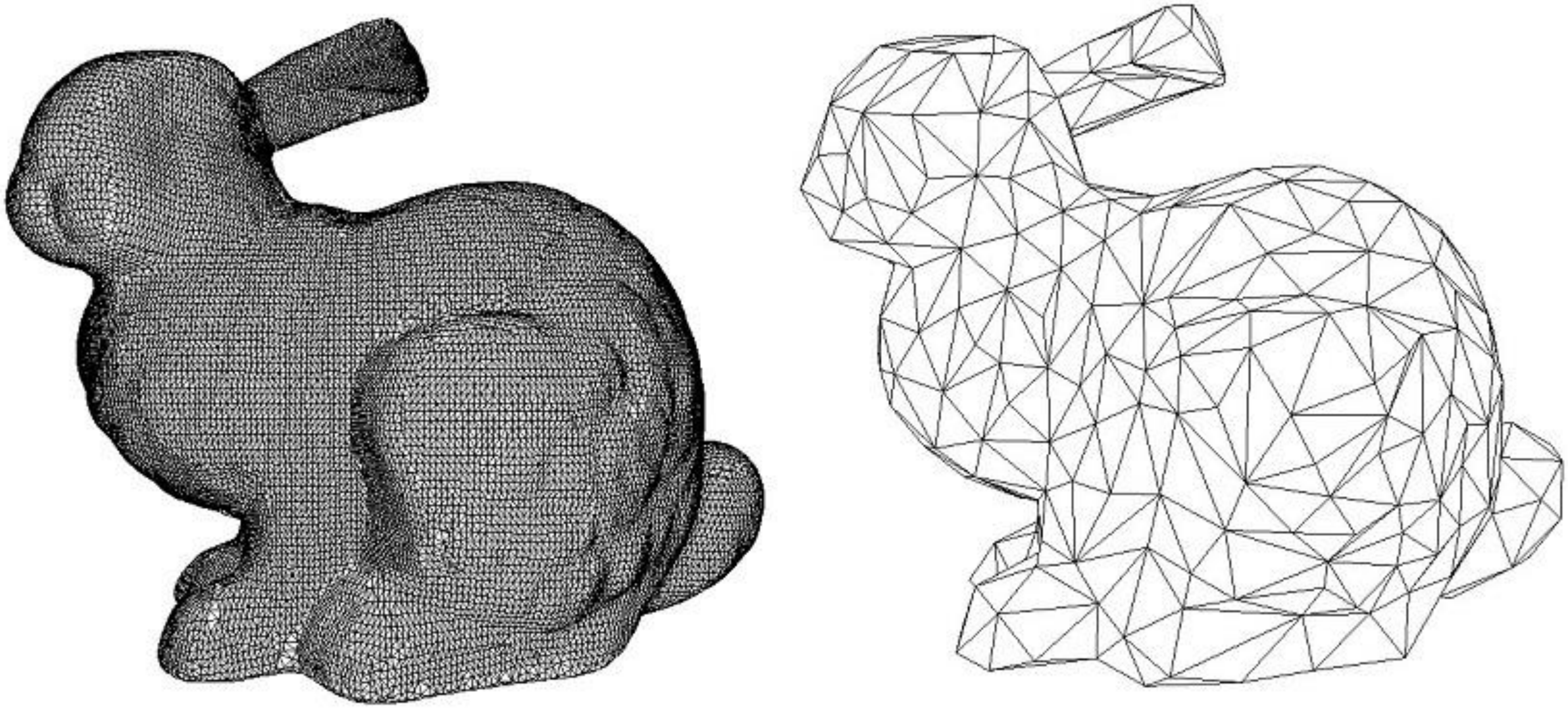
Surface Simplification



[2]

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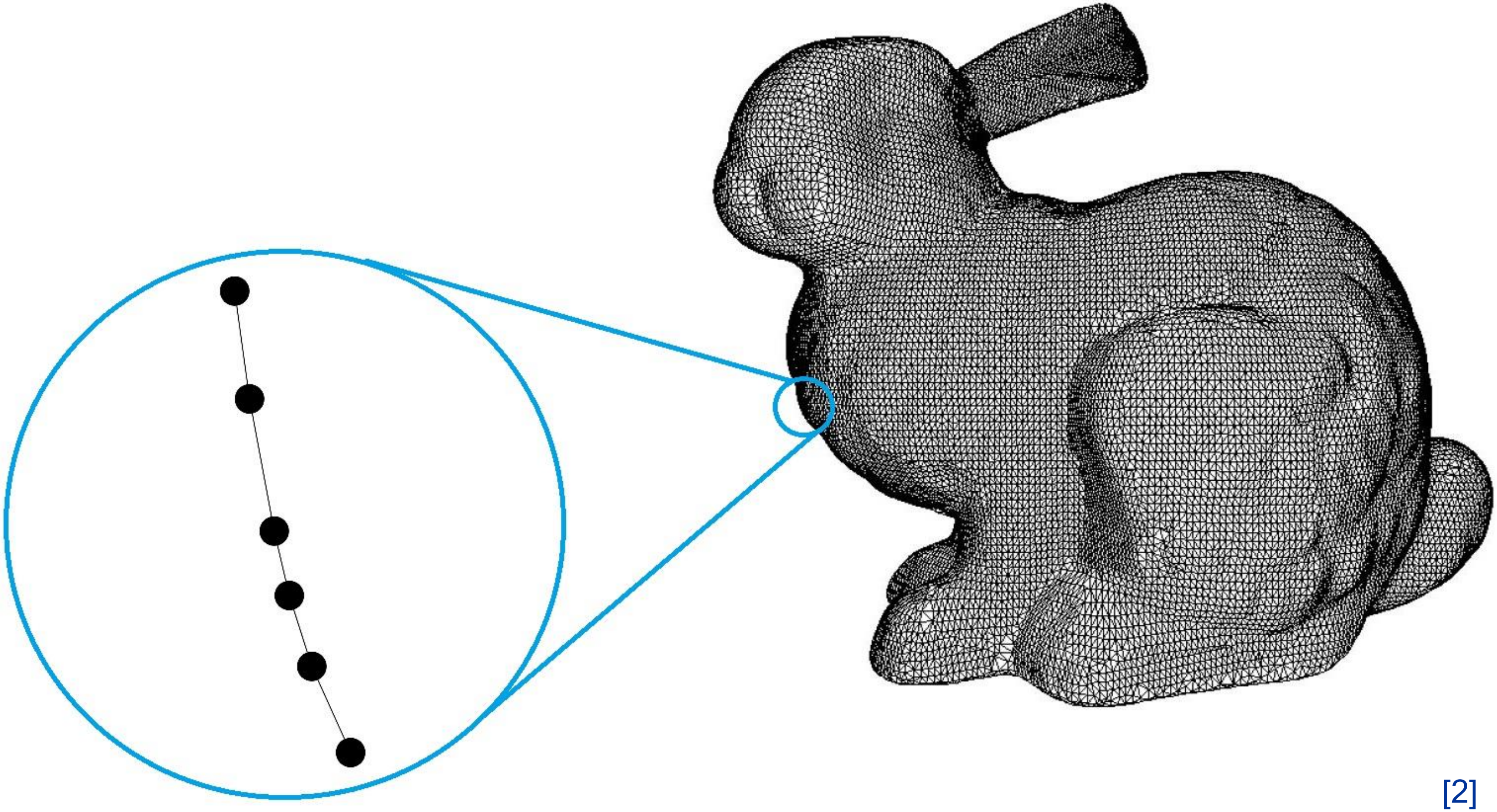
Pair Contraction



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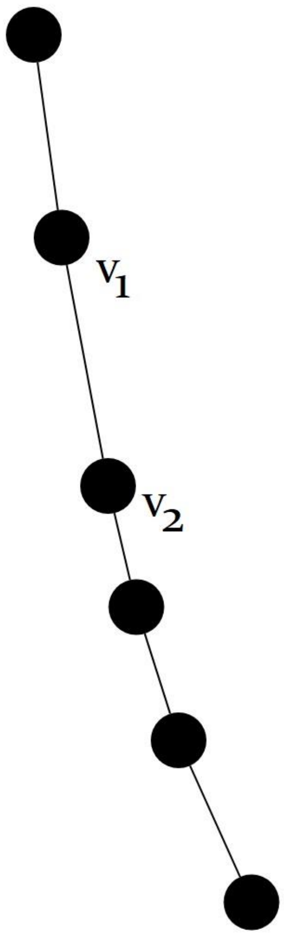
Pair Contraction



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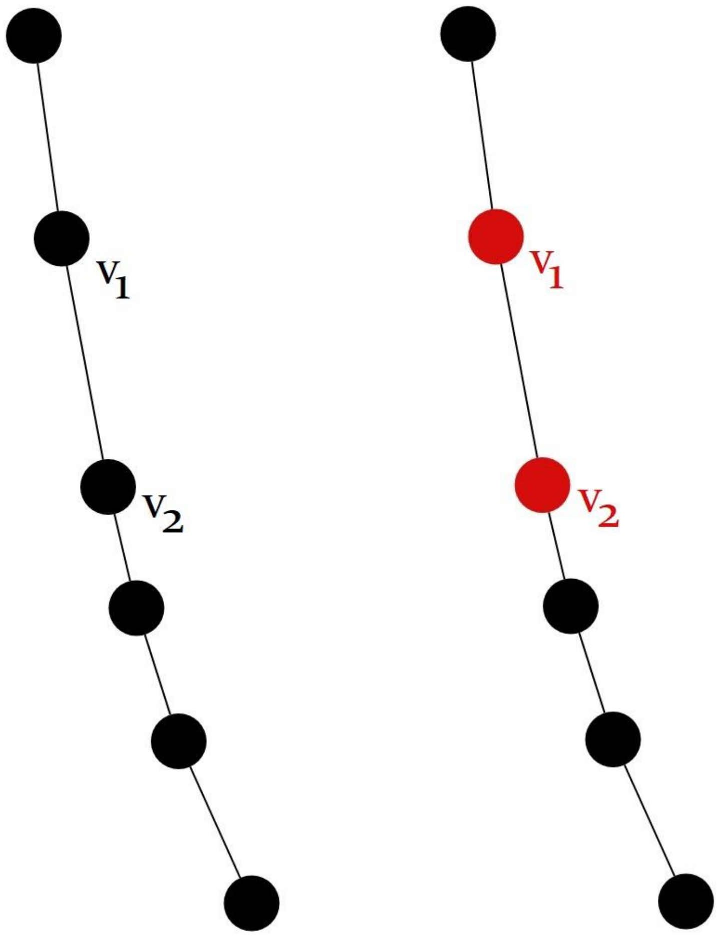

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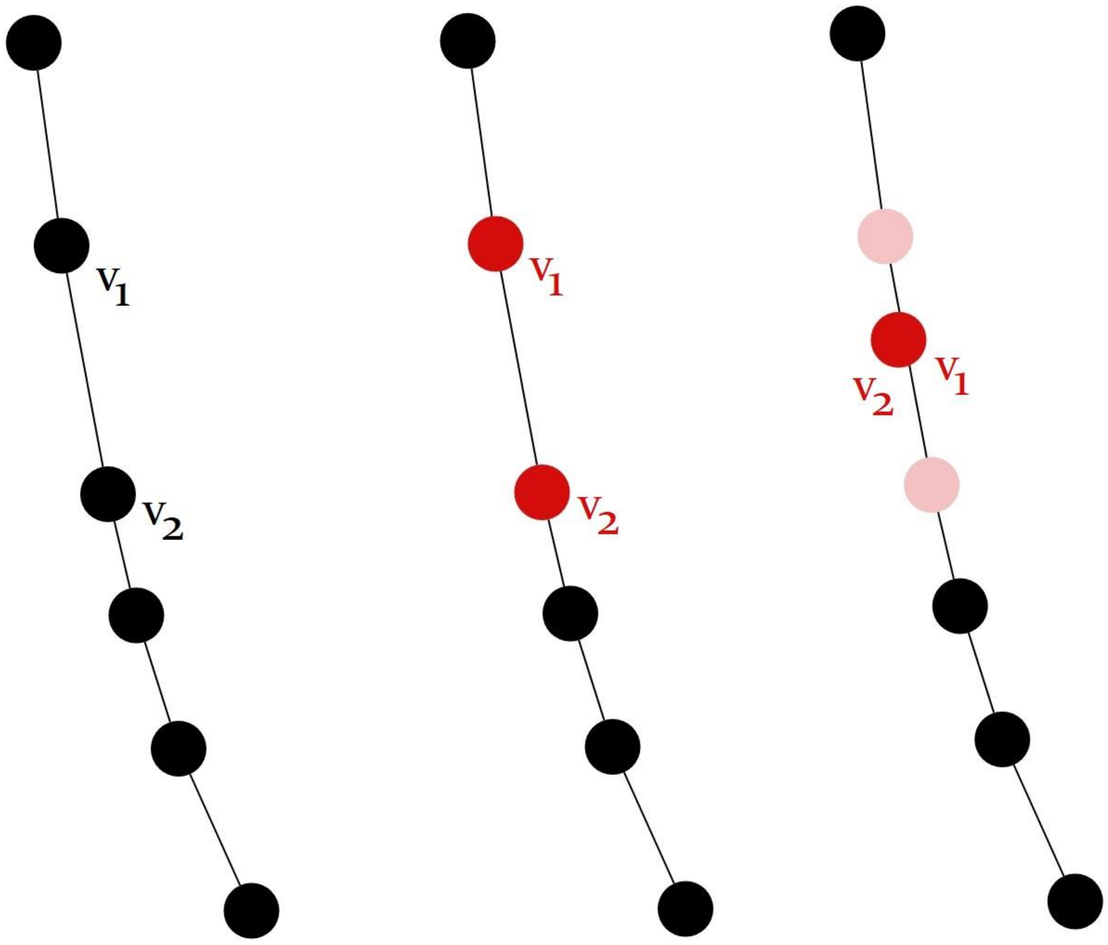

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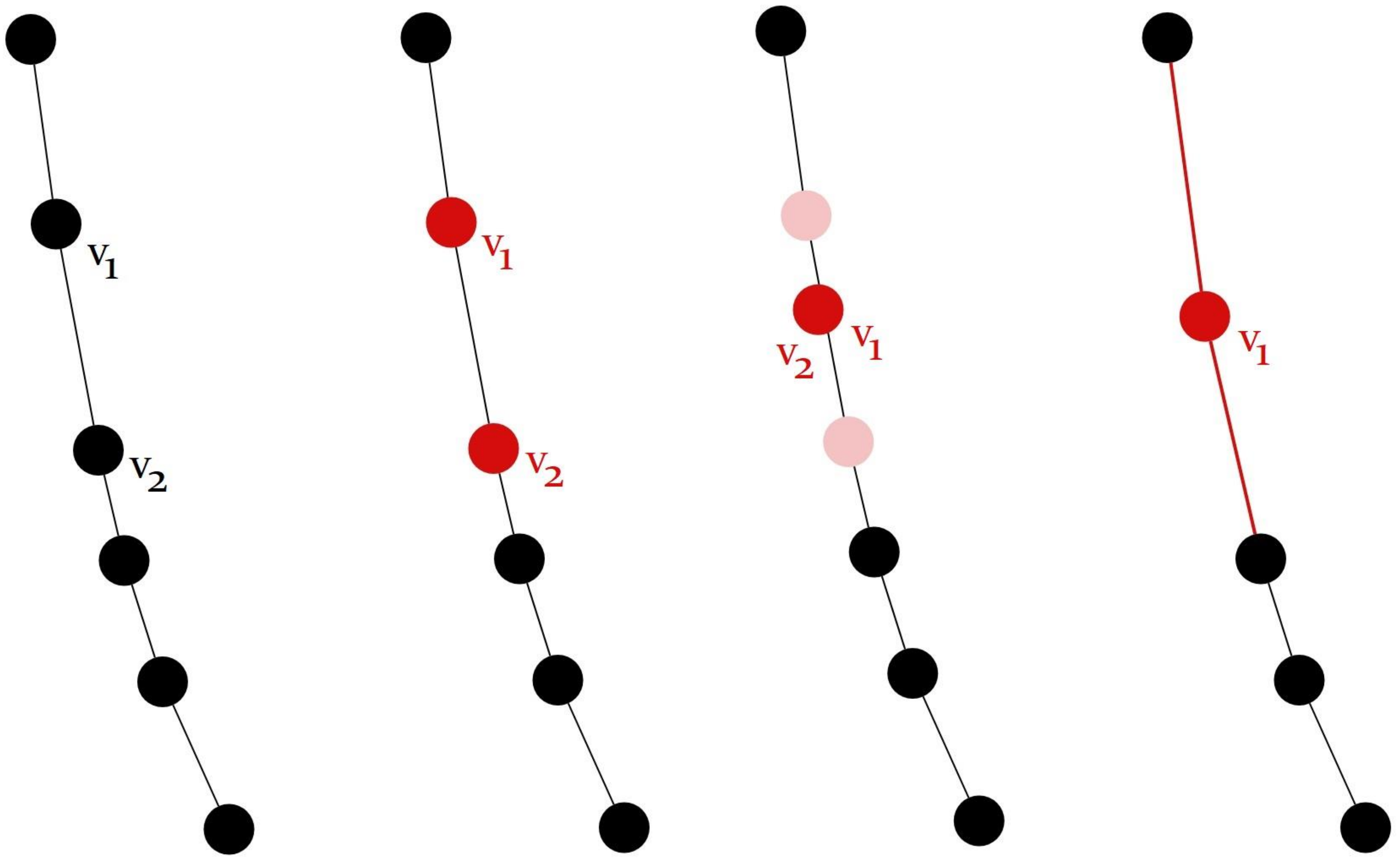

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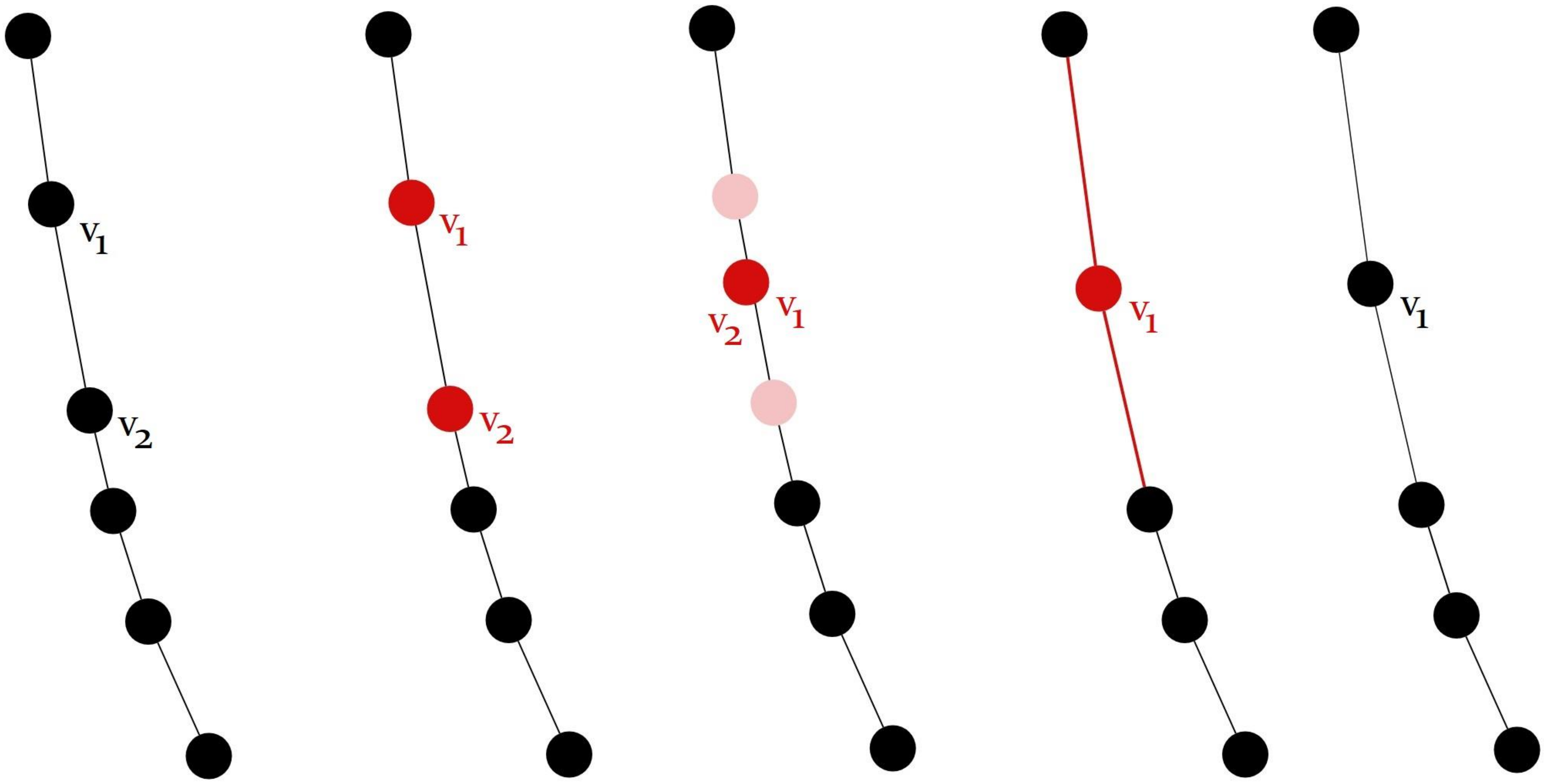

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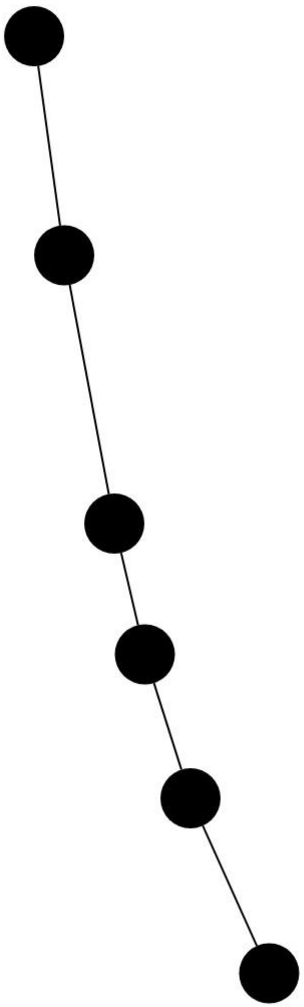

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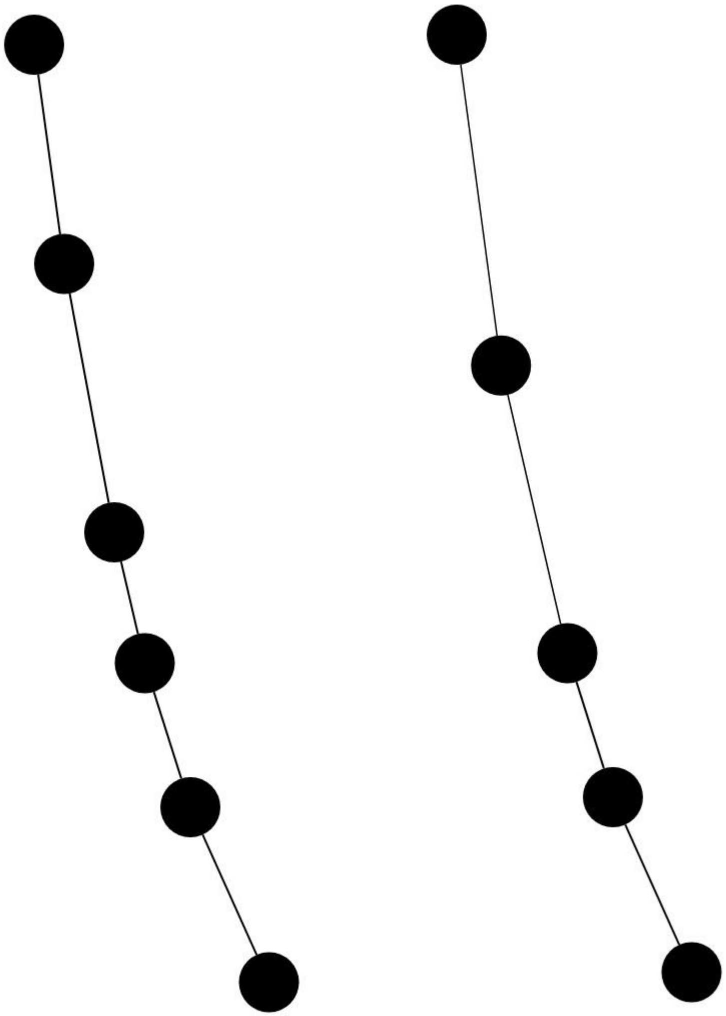

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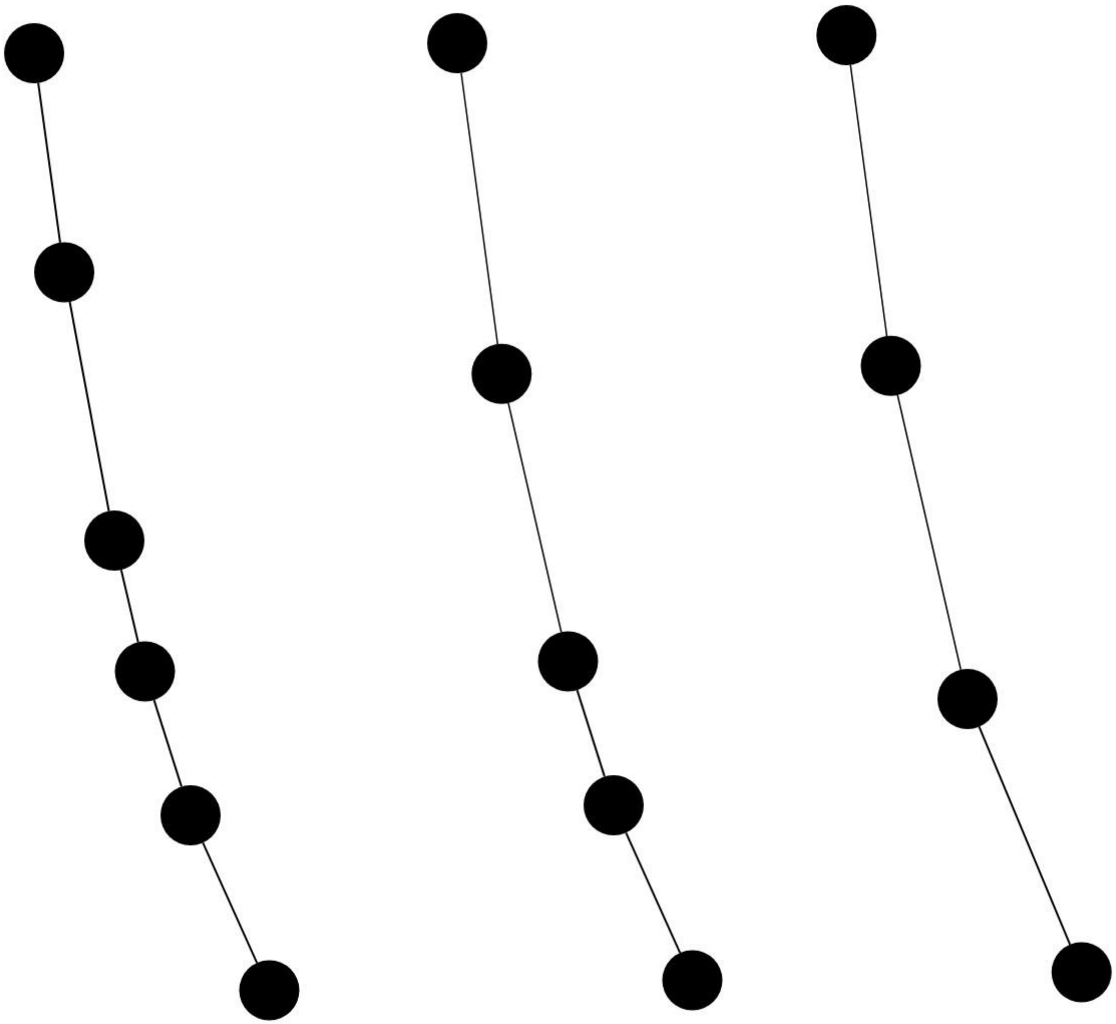

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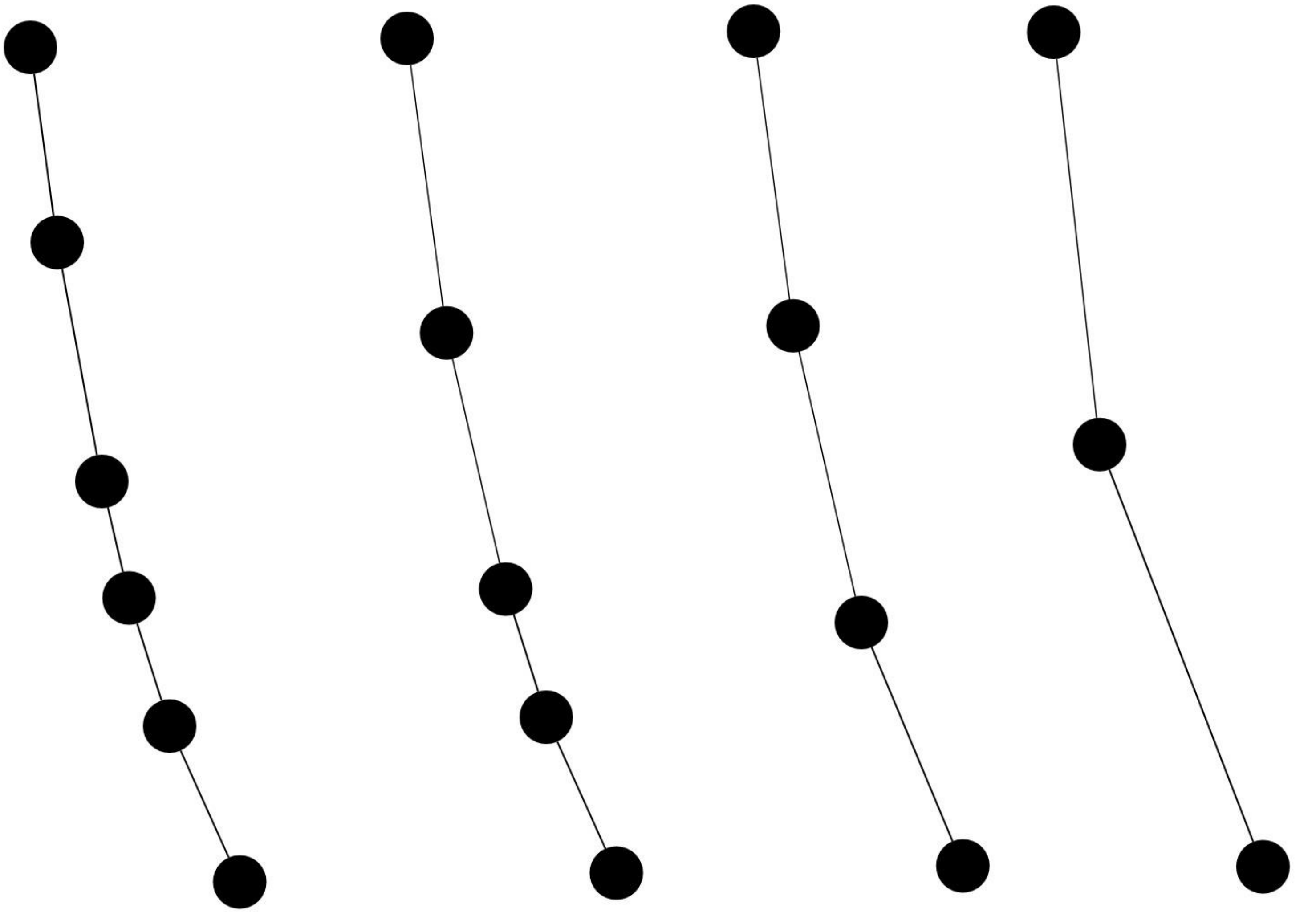

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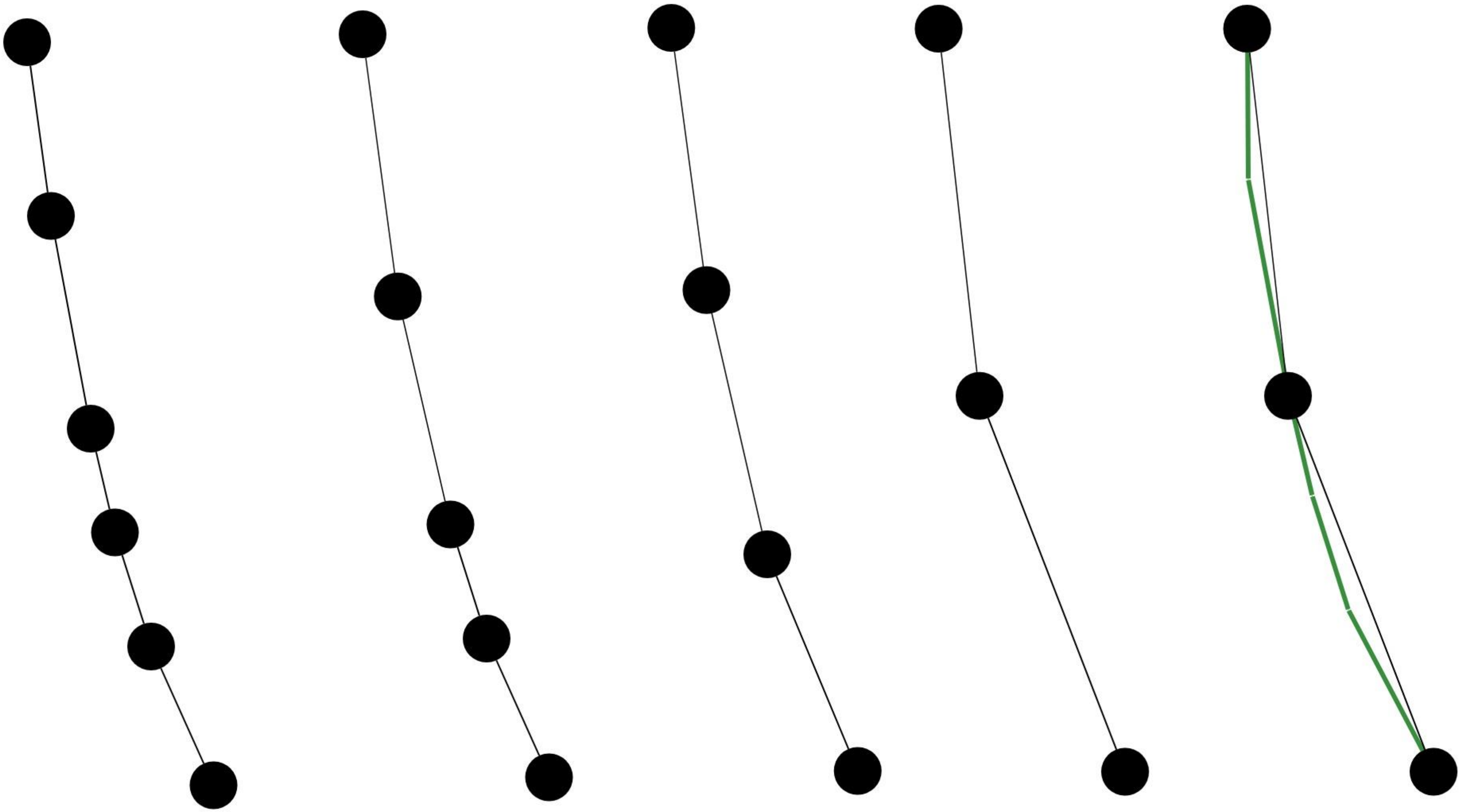

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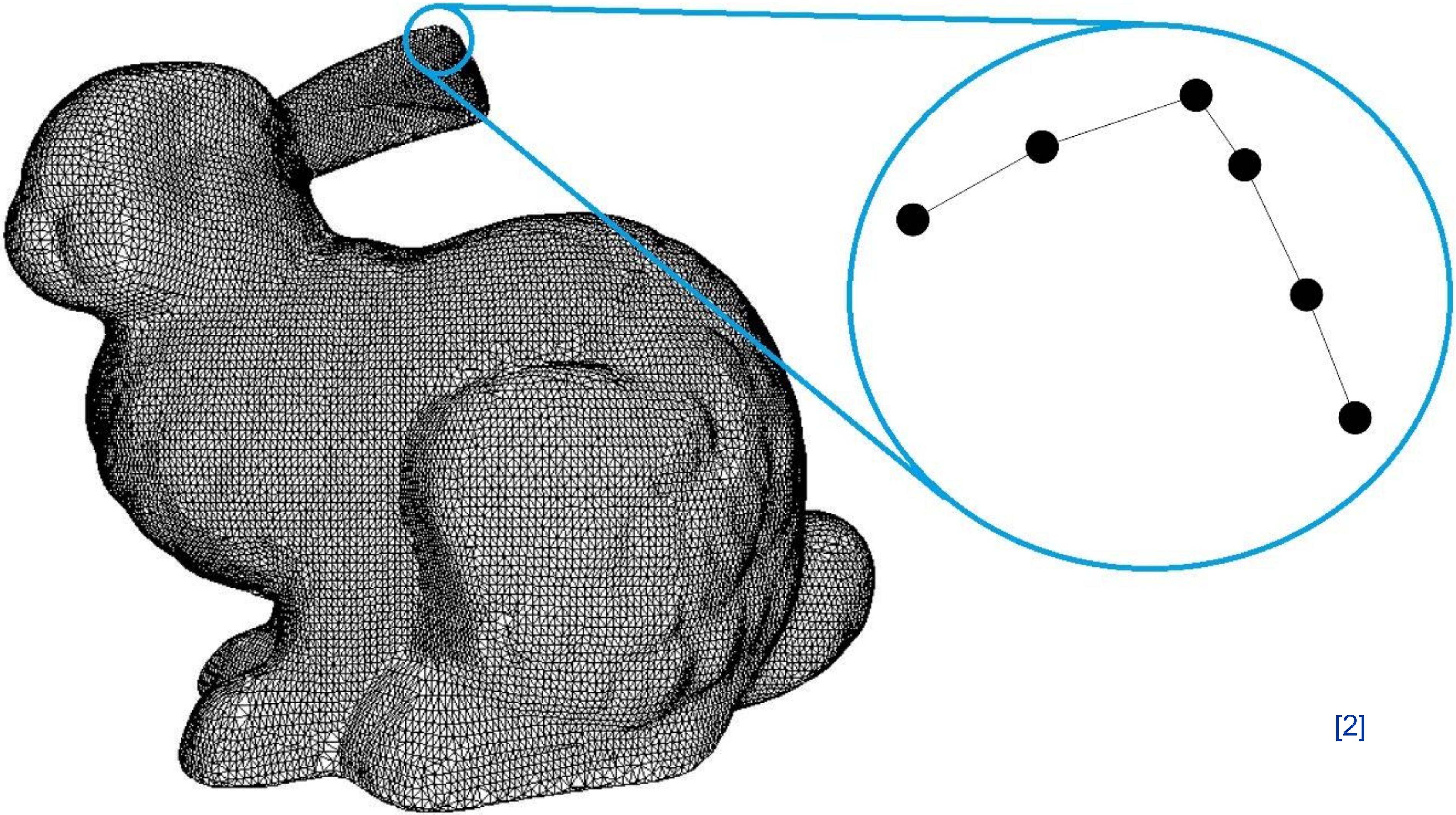

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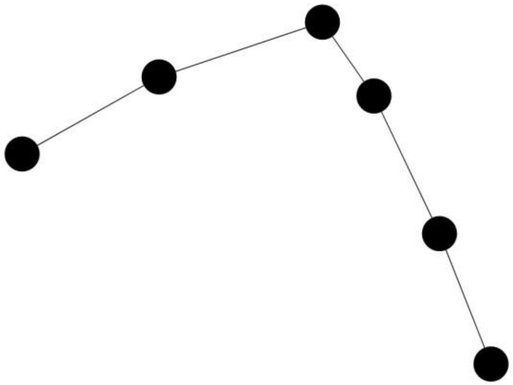

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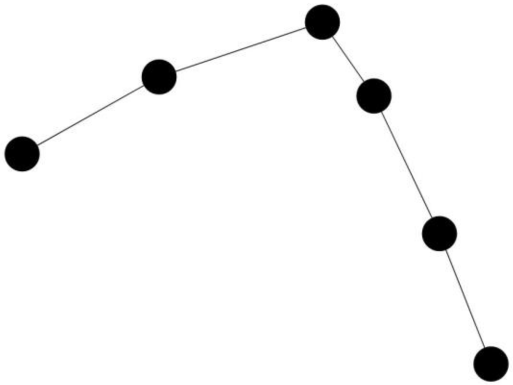

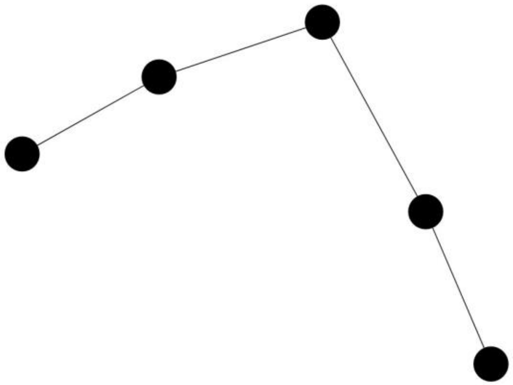

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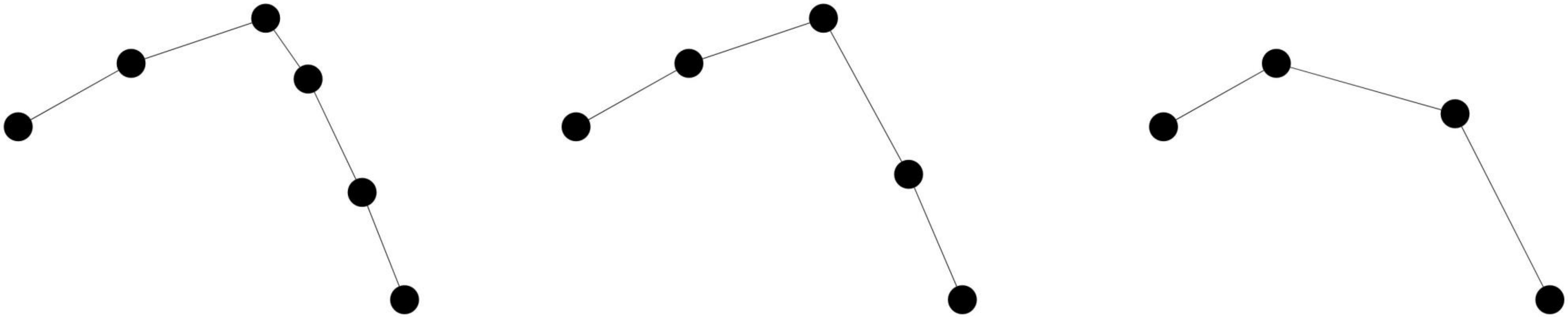

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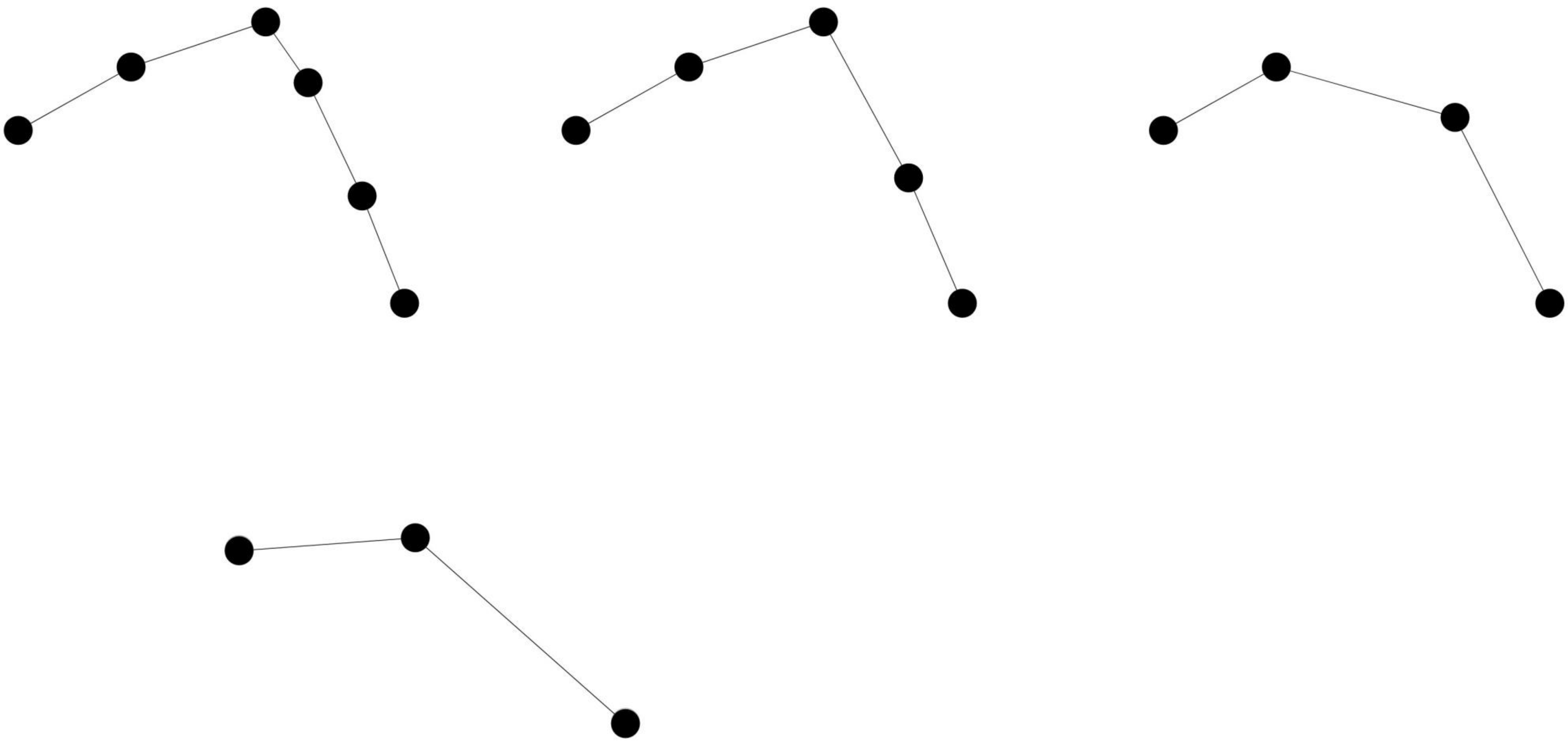

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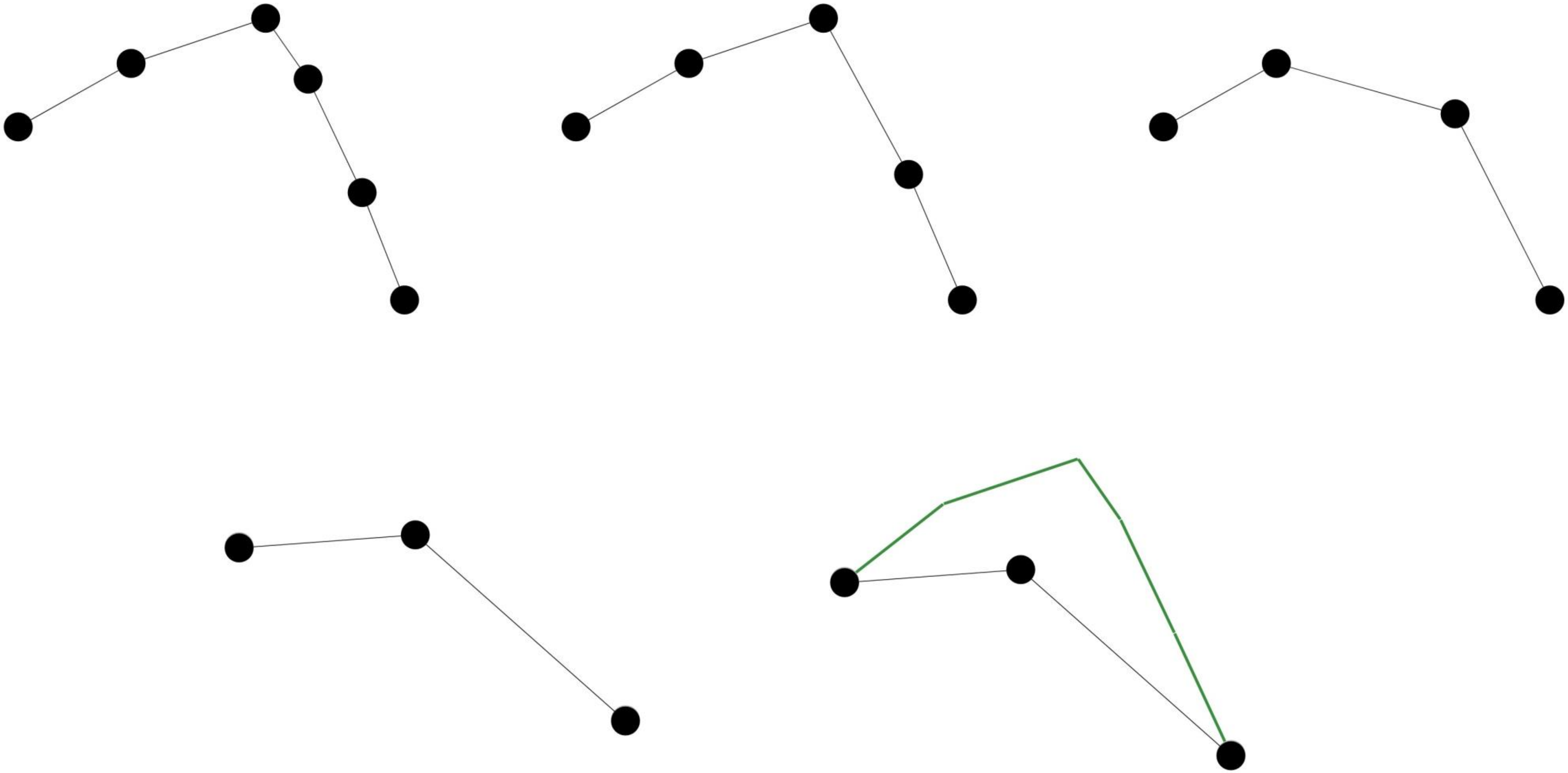

Pair Contraction – Summary

- Movement of two vertices to a new position
- Reassignment of edges
- Deletion of second vertex

[2]

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1. Surface Simplification
2. Pair Contraction
3. Quadric Error Metric
4. Contraction Target
5. Algorithm

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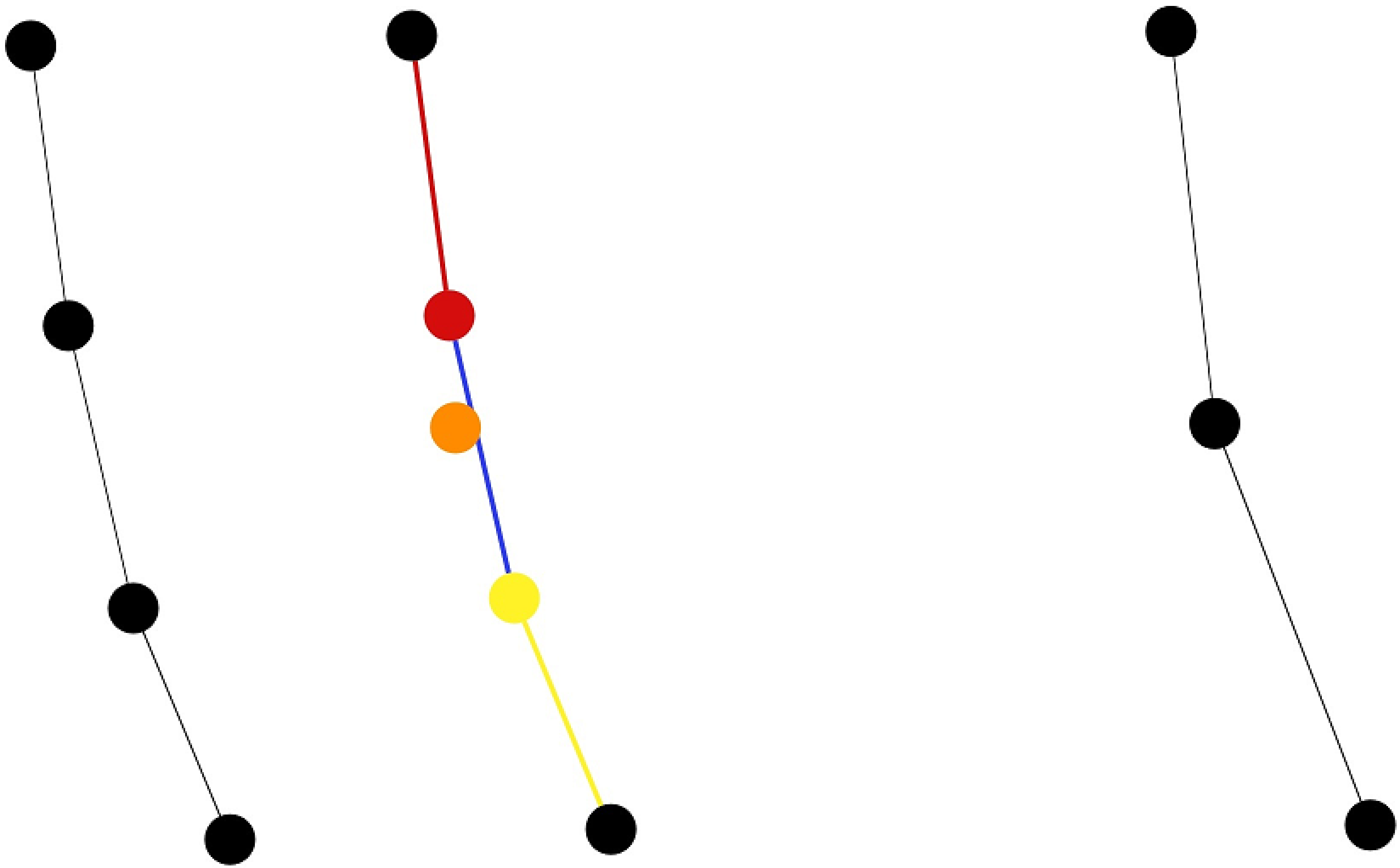
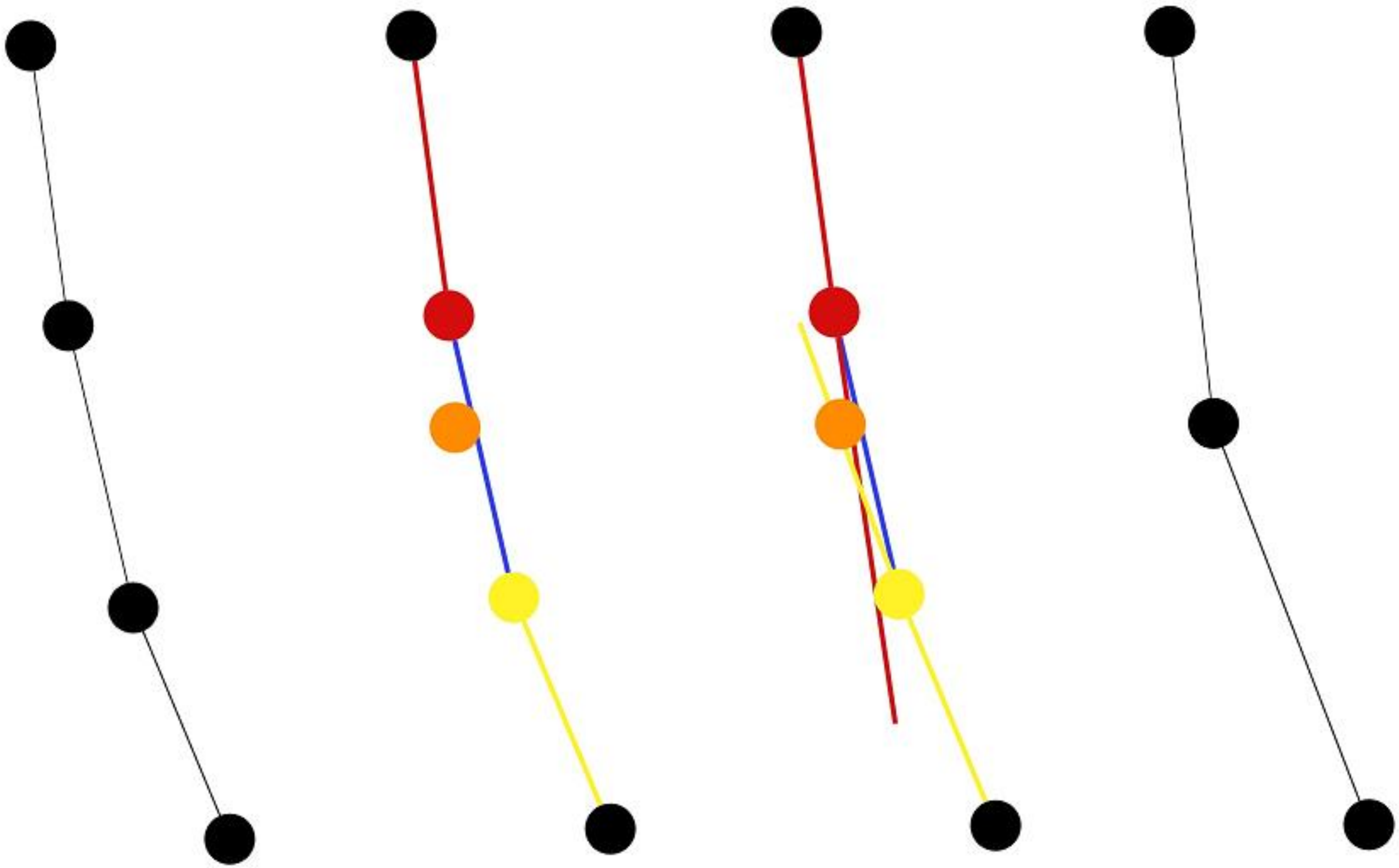
Quadric Error Metric: Plane Intersection

Quadric Error Metric: Plane Intersection



Quadric Error Metric: Plane Intersection



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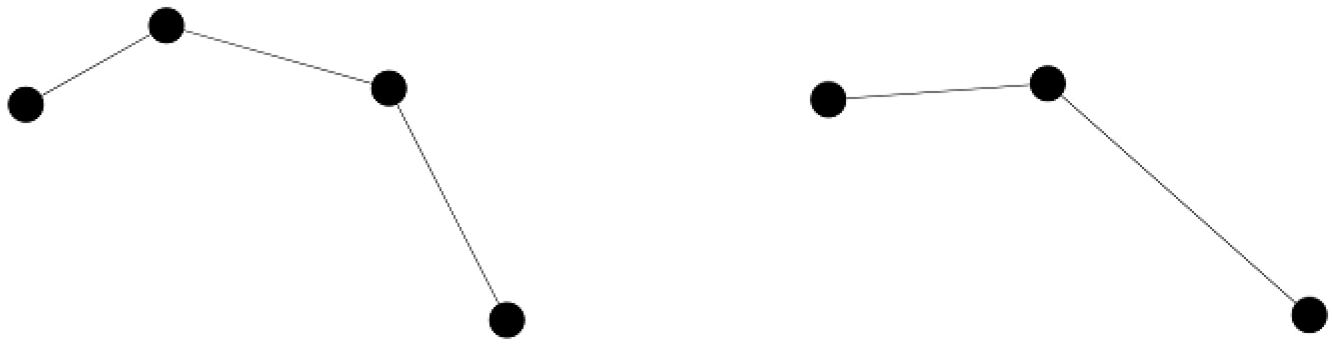

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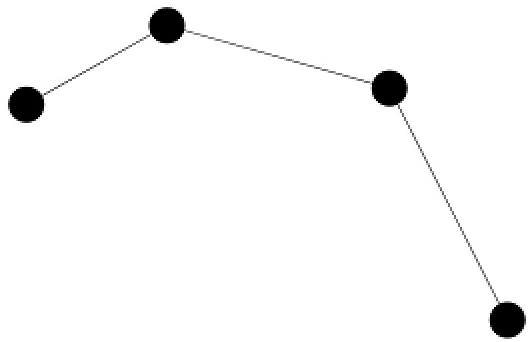
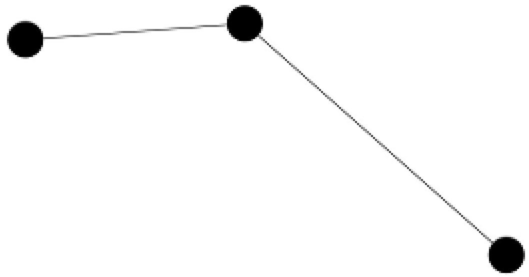
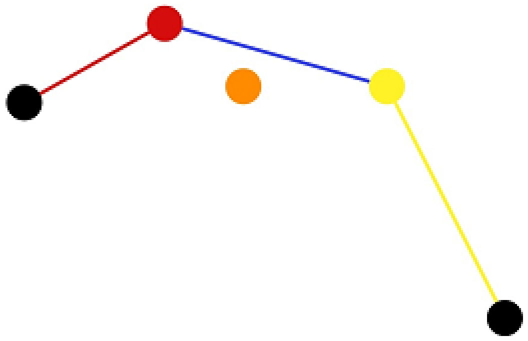
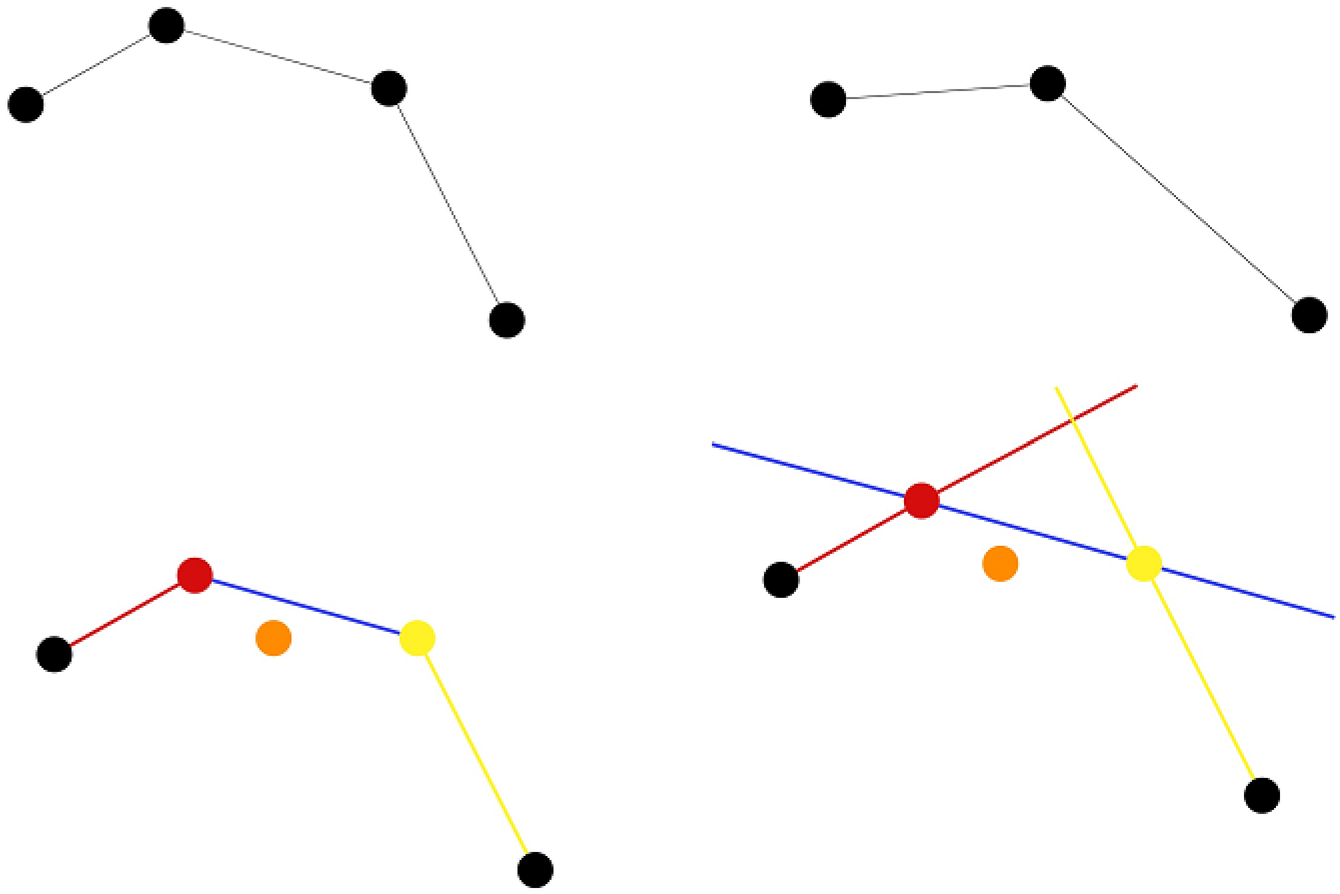






Quadric Error Metric: Plane Intersection



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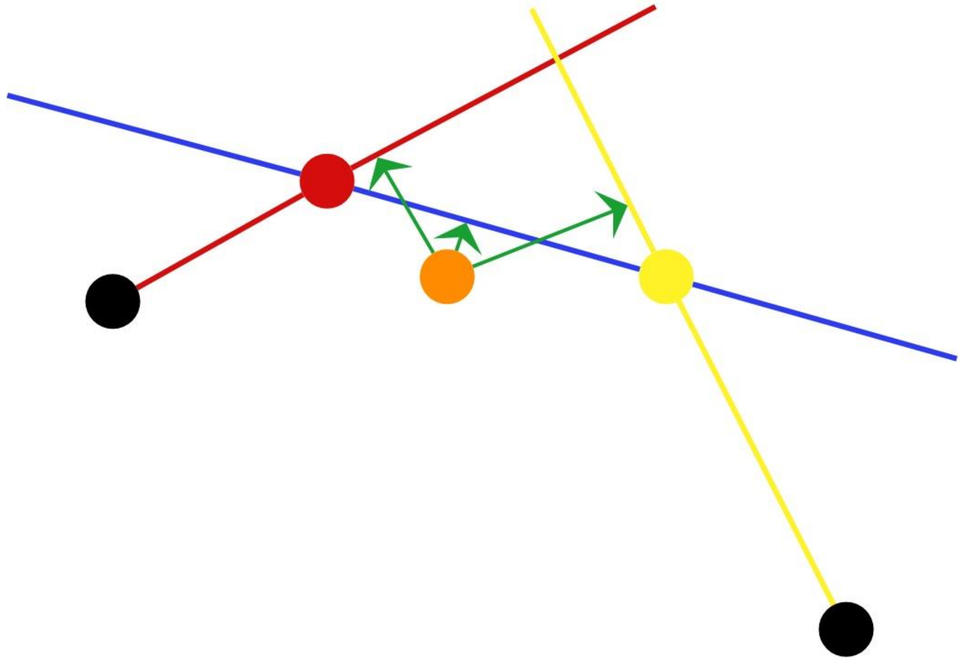
1. Surface Simplification
2. Pair Contraction
3. Quadric Error Metric
    1. Plane Intersection
    2. Squared Distances
4. Contraction Target
5. Algorithm

Quadric Error Metric: Squared Distances

- Quadric Error Metric as quality prediction of a contraction

- Usage of squared distances to planes



Quadric Error Metric: Squared Distances

- plane p: $ax + by + cz + d = 0$ with $a^2 + b^2 + c^2 = 1$
- vertex $v = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$
- distance of p and v = $\frac{|ax+by+cz+d|}{\sqrt{a^2+b^2+c^2}}$ (Cheney, 2010)
- squared distance $= \frac{(ax+by+cz+d)^2}{a^2+b^2+c^2} = (ax + by + cz + d)^2$

Quadric Error Metric: Squared Distances

- squared distance $= (ax + by + cz + d)^2$

$$= \left((a, b, c, d) * \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \right)^2$$

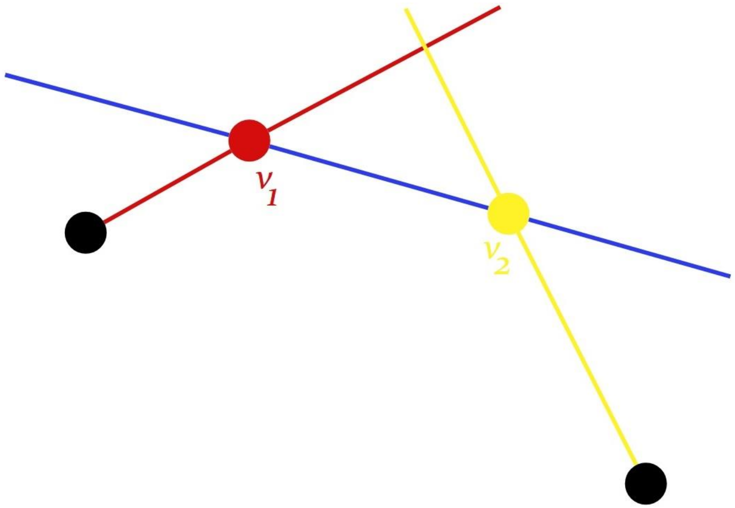
$$= (x, y, z, 1) * \begin{pmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{pmatrix} * \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$= v^T * \begin{pmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{pmatrix} * v$$

[1]

Quadric Error Metric: Squared Distances

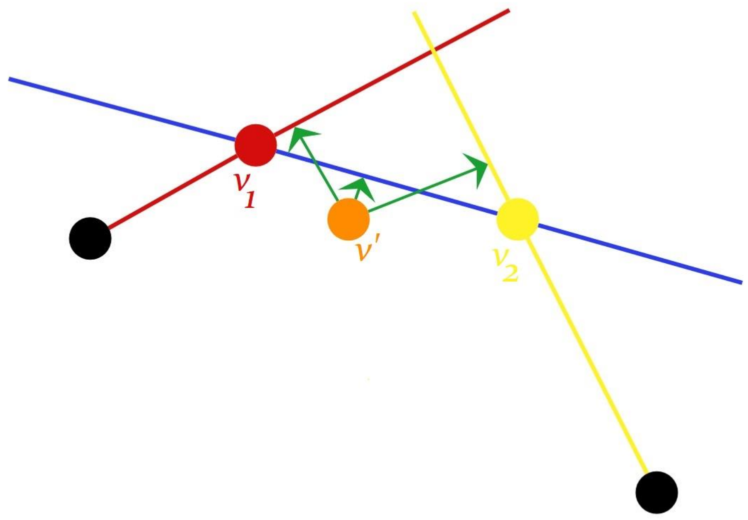
- $Q(v_1) = \begin{pmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{pmatrix} + \begin{pmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{pmatrix}$



- $Q(v_2) = \begin{pmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{pmatrix} + \begin{pmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{pmatrix}$

Quadric Error Metric: Squared Distances

- $Q(v') = Q(v_1) + Q(v_2)$



- Error at v': $\Delta(v') = v'^T Q(v') \, v'$ [1]

Squared Distances – Summary

- Quadric Error Metric predicts the error of a contraction
- Error $\Delta(v') = v'^T Q \, v'$ [1]
- Q-Matrix as sum of squared distances to the planes from the contracted vertices

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1. Surface Simplification
2. Pair Contraction
3. Quadric Error Metric
4. Contraction Target
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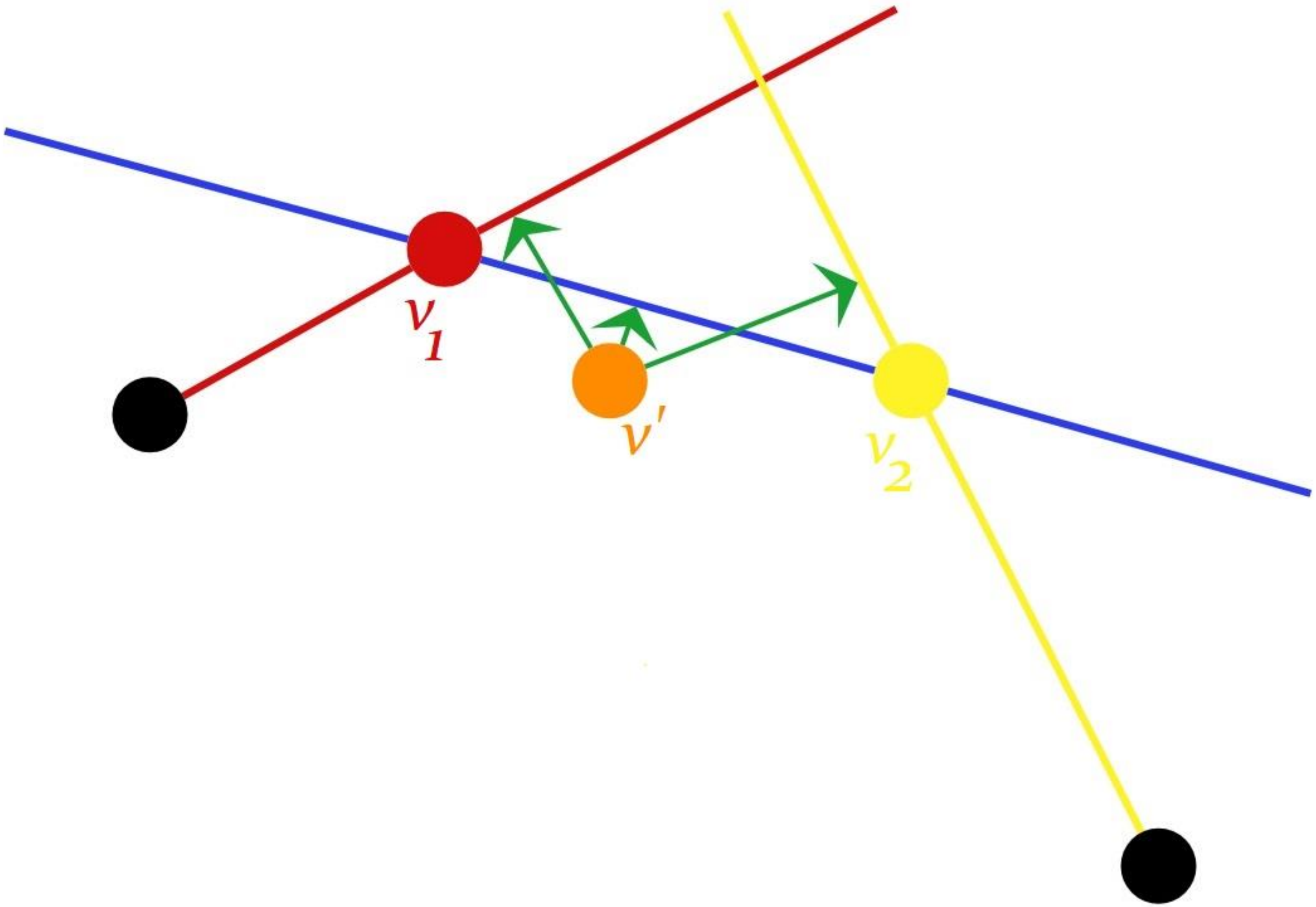
Contraction Target



Contraction Target

- location for the contracted vertex
- minimize the error
- Δ at vertex v':

$$\Delta(v') = v'^T Q v' = (x, y, z, 1) \begin{pmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{12} & q_{22} & q_{23} & q_{24} \\ q_{13} & q_{23} & q_{33} & q_{34} \\ q_{14} & q_{24} & q_{34} & q_{44} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Contraction Target

- $\Delta(v) = q_{11}x^2 + 2q_{12}xy + 2q_{13}xz + 2q_{14}x + q_{22}y^2 + 2q_{23}yz + 2q_{24}y + q_{33}z^2 + 2q_{34}z + q_{44}$

[1]

- calculate partial derivatives of the error and set them to 0
  - $\frac{\delta\Delta}{\delta x} = q_{11}x + q_{12}y + q_{13}z + q_{14} = 0$
  - $\frac{\delta\Delta}{\delta y} = q_{12}x + q_{22}y + q_{23}z + q_{24} = 0$
  - $\frac{\delta\Delta}{\delta z} = q_{13}x + q_{23}y + q_{33}z + q_{34} = 0$

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Contraction Target

- these conditions can be written as:

$$\begin{pmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{12} & q_{22} & q_{23} & q_{24} \\ q_{13} & q_{23} & q_{33} & q_{34} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

- inverting this matrix yields the contraction target v'

$$v' = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{12} & q_{22} & q_{23} & q_{24} \\ q_{13} & q_{23} & q_{33} & q_{34} \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

[1]

Contraction Target - Summary

- error minimization by choosing the optimal target location
- calculation ‘only‘ requires inverting a matrix
- contraction target is the last column of the inverted, modified matrix

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5. Algorithm
    1. Initial Matrices
    2. Pair Selection
    3. Contraction Targets
    4. Error
    5. Pair Contraction

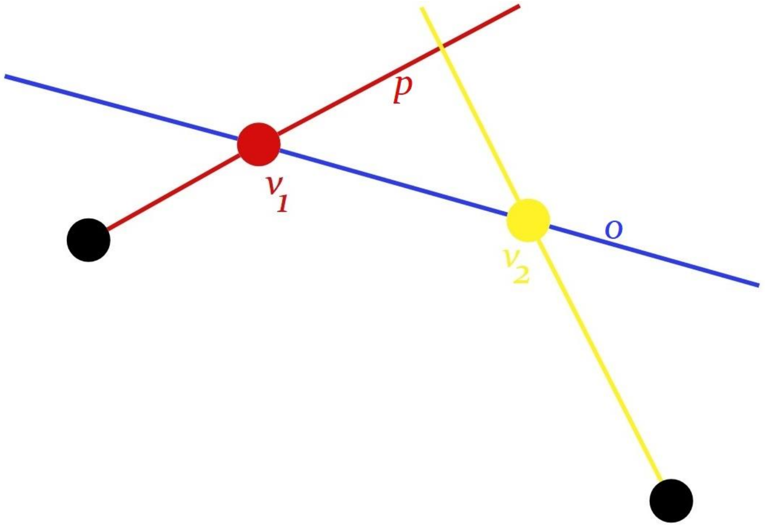
Algorithm – Initial Matrices

- Compute Q-Matrices for all vertices.

- $$Q(v_1) = \begin{pmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{pmatrix} + \begin{pmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{pmatrix}$$

with plane p: $ax + by + cz + d = 0$

and plane o: $ax + by + cz + d = 0$



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5. Algorithm
    1. Initial Matrices
    2. Pair Selection
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Algorithm – Pair Selection

- Selection of all pairs of vertices, that:
  - share an edge or
  - are close to each other
- Calculation of new Q-Matrices

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5. Algorithm
    1. Initial Matrices
    2. Pair Selection
    3. Contraction Targets
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Algorithm – Contraction Targets

- Calculation of the contraction targets v' for all pairs

$$v' = \begin{pmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{12} & q_{22} & q_{23} & q_{24} \\ q_{13} & q_{23} & q_{33} & q_{34} \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

[1]

Content

5. Algorithm
    1. Initial Matrices
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Algorithm - Error

- Calculation of the error of all possible contractions

$$\Delta(v') = v'^T \, Q \, v'$$

[1]

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Algorithm – Pair Contraction

- Contraction of the pair with smallest error
  - Movement of vertices to the contraction target
  - Reassignment of edges
  - Deletion of second vertex
  - Error update for affected pairs
  - Repeat

[2]

Algorithm - Summary

1. Initial Matrices
2. Pair Selection
3. Contraction Targets
4. Error
5. Pair Contraction

Sources

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Surface Simplification Using Quadric Error Metrics.
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