Computer Graphics
Material Point Method MPM

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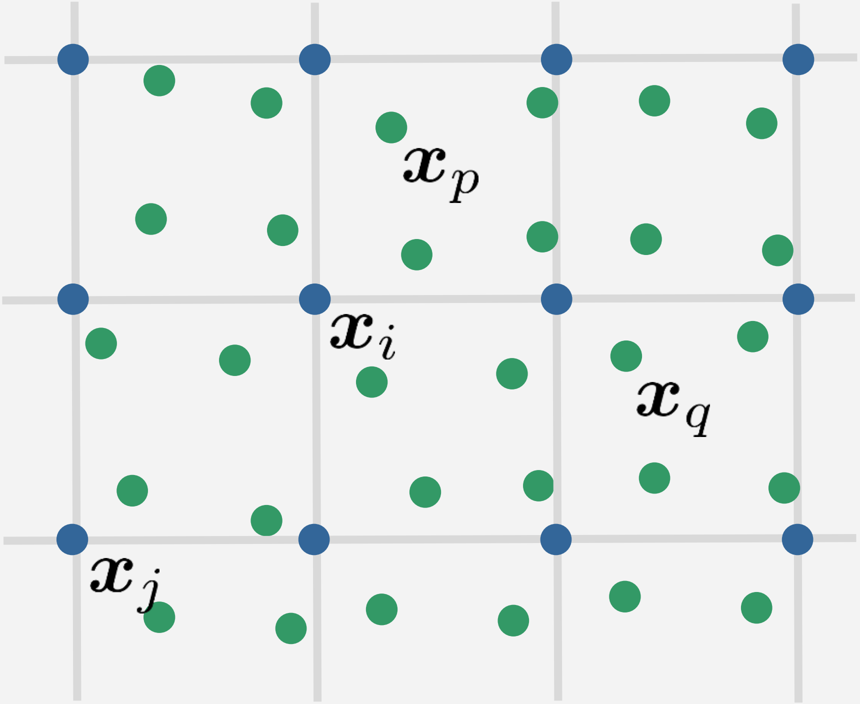
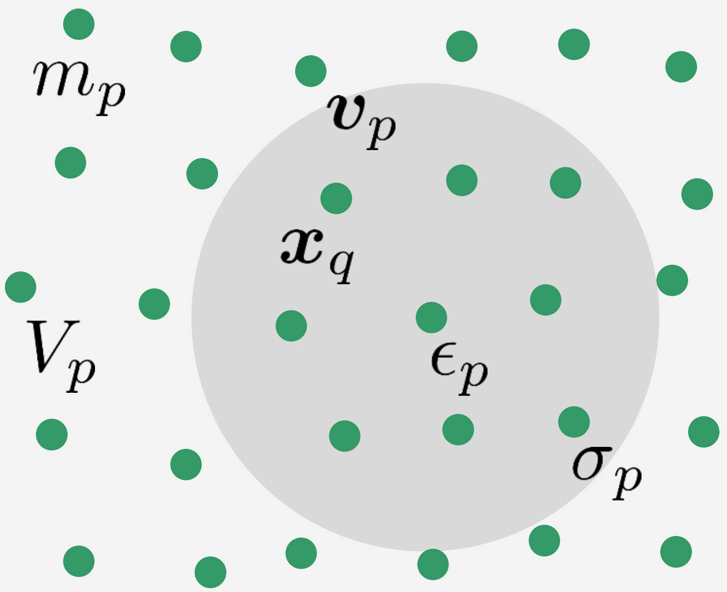
Two Sample Sets

– Material points p, q and grid nodes i, j



Initialization

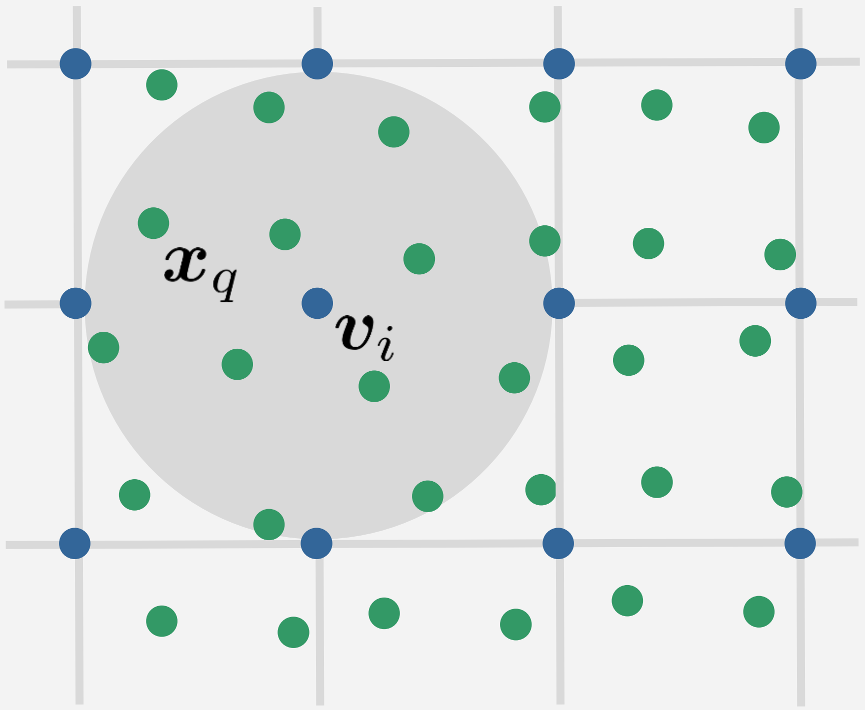


Mass, volume, velocity, strain, stress
are considered at material points.

E.g.: $$\epsilon_p = \frac{V_p^0}{V_p} - 1 = V_p^0 \sum_q W_{pq} - 1$$

$$\sigma_p = k\epsilon_p$$ *W* is a shape function.

1. Velocity Interpolation at Grid Nodes



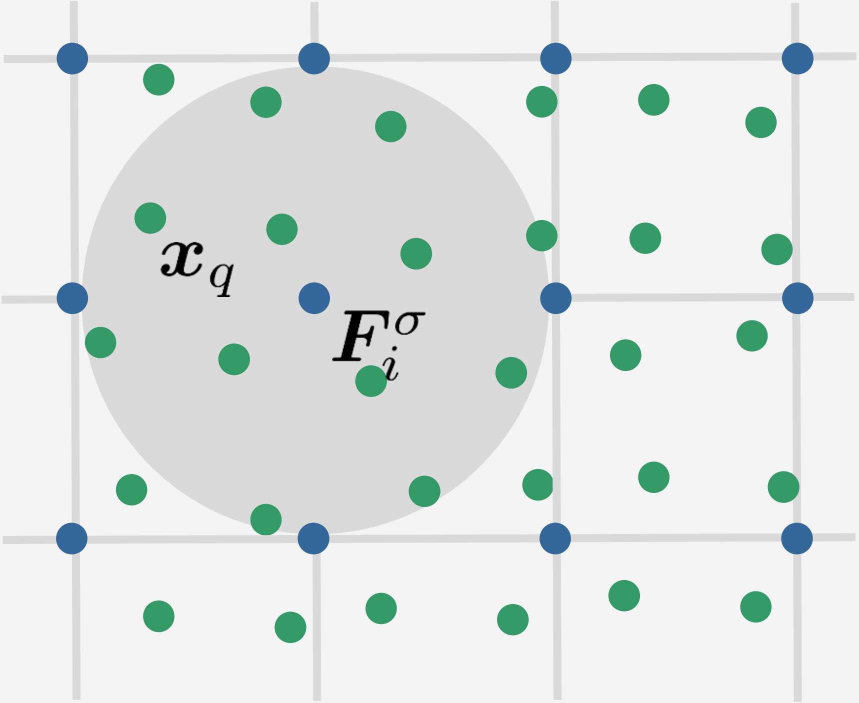
Velocities and masses are interpolated at grid nodes.

$$m_i = \sum_q m_q V_q W_{iq} \quad \boldsymbol{v}_i = \sum_q \boldsymbol{v}_q V_q W_{iq}$$

2. Velocity Update at Grid Nodes

E.g., gravity and boundary handling.

$$\boldsymbol{v}_i = \boldsymbol{v}_i + \Delta t \boldsymbol{a}_i^{\text{other}}$$

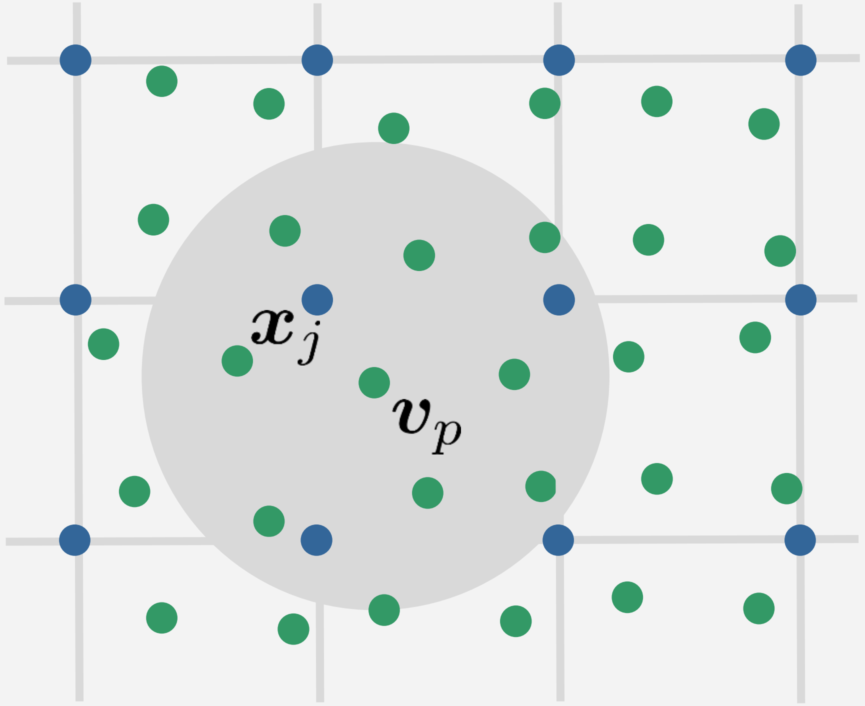


Acceleration from stress

$$\boldsymbol{F}_i^\sigma = -V_i \nabla \cdot \sigma_i = -V_i \sum_q \sigma_q V_q \nabla W_{iq}$$

$$\boldsymbol{v}_i = \boldsymbol{v}_i + \Delta t \frac{1}{m_i} \boldsymbol{F}_i^\sigma$$

3. Velocity Interpolation at Material Points



Note! The kernel support is actually at least twice as large as shown in the illustration.

Velocities are interpolated at material points.

$$\boldsymbol{v}_p = \sum_j \boldsymbol{v}_j V_j W_{pj} \qquad \text{PIC}$$

or

$$\boldsymbol{v}_p = \boldsymbol{v}_p + \Delta t \sum_j (\boldsymbol{a}_j^{\text{other}} + \boldsymbol{a}_j^{\sigma}) V_j W_{pj} \qquad \text{FLIP}$$

4. Stress Update at Material Points

$$\epsilon_p = \epsilon_p + \Delta t \nabla \cdot \boldsymbol{v}_p$$

$$= \epsilon_p + \Delta t \sum_j \boldsymbol{v}_j V_j \nabla W_{pj}$$

$$\sigma_p = k\epsilon_p$$

Again: Use grid node neighbors instead of particle neighbors.

5. Advect Particles and Go To 1.

$$\boldsymbol{x}_p = \boldsymbol{x}_p + \Delta t \boldsymbol{v}_p$$

Plus some boundary handling.

Properties

- Particles never use particle neighbors (except in the initialization)
- Strain and Stress are generally 3x3 matrices for complex materials instead of scalars
- MPM often prefers an alternative approach to compute stress

$\boldsymbol{F}$	$\rightarrow$	Ψ	$\rightarrow$	$\boldsymbol{P}$
Deformation gradient		Deformation energy (potential energy, elastic energy)		First Piola-Kirchhoff Stress (in contrast to Cauchy stress σ)

- Acceleration is often expressed with the deformation gradient